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CCEA GCSE

MATHEMATICS

M2 M6

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Contents

See page 5 for information on how to obtain the Answers.

Introduction

This book covers the Foundation Tier Option 2 (M2 and M6) pathway of the CCEA specification for Mathematics (first teaching in September 2017). Specifically, it covers:

- the material required at level M2 and level M6, and
- the material required at M1 and M5 at a depth that is appropriate for an M2 and M6 student.

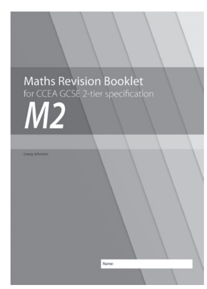
This book has undergone a detailed quality assurance check by experienced mathematician Joe McGurk prior to publication.

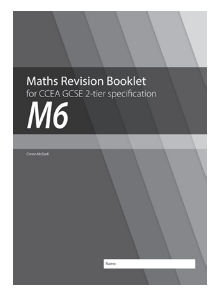
Feedback from teachers is that highlighting the different units in a Mathematics textbook is overly complex and potentially confusing. As a result, this book has been deliberately designed to cover the course without making any reference to specific units within the text.

Teachers and students can be assured that, if they are on the M2 and M6 pathway, then they simply need to teach or study the whole contents of this textbook. The authors have chosen a thematic approach, where similar material that appears in both M2 and M6 is treated together in the book. Thus, while the first part of the book generally covers M2 material, and the second part focuses on M6 material, there are some chapters that contains both M2 and M6 material.

The book also contains eight 'progress review' sections which act as checkpoints for students. Students can use these to assess their understanding of material at regular intervals throughout the course.

Finally, the book is supplemented by two Revision Booklets, one for M2 and one for M6, which students can use to prepare for the exam. These are two of eight books that Colourpoint publishes for M1 to M8. They are available from Colourpoint.





Answers

The answers to all the exercises are available in PDF format and can be downloaded from our web site. Go to www.colourpointeducational.com and search the title of this book. Once you reach the page about this book, you will find a a link to download the answers.

If you have any problems, please contact Colourpoint directly. Our contact details are on page 2.

Chapter 1
Working With Integers

1.1 Introduction

This chapter is about **integers**. An integer is a whole number that can be positive, negative, or zero. Examples of integers are: –5, 0, 1, 92, and 2149. The chapter covers:

- Negative numbers.
- Factors and multiples.
- Prime numbers.
- Squares and square roots.
- Cubes and cube roots.
- Index notation.

Before you start you should:

- Understand that there are positive and negative numbers.
- Know and understand the following terms: factor, multiple, common factor, common multiple, prime, square, positive and negative square root, cube and cube root.
- Know that 4^2 means 4×4 and that 4^3 means $4 \times 4 \times 4$.

In this chapter you will learn how to:

- Work with indices.
- Find a prime factorisation for any positive whole number.
- Find the Highest Common Factor for two positive whole numbers.
- Find the Lowest Common Multiple for two positive whole numbers.

1.2 Negative Numbers

You should understand how to use the four arithmetic operations (addition, subtraction, multiplication and division) using both positive and negative numbers.

Example 1

Work out the following:

(a) $-4 + -2$ **(b)** $-10 - -3$ **(c)** $-7 + 5$ **(d)** $-6 - 5$ **(e)** $8 - -10$
(f) -2×-9 **(g)** 3×-7 **(h)** $-20 \div 5$ **(i)** $-36 \div -6$

(a) Adding a negative number is the same as subtracting the related positive number, so:
$-4 + -2$
$= -4 - 2$
$= -6$

(b) Subtracting a negative number is the same as adding the related positive number, so:
$-10 - -3$
$= -10 + 3$
$= -7$

(c) $-7 + 5 = -2$ **(d)** $-6 - 5 = -11$ **(e)** $8 - -10 = 8 + 10 = 18$

(f) Multiplying or dividing two negative numbers results in a positive answer:
-2×-9
$= 18$

(g) Multiplying or dividing a negative number by a positive number results in a negative answer:
3×-7
$= -21$

(h) $-20 \div 5 = -4$ **(i)** $-36 \div -6 = 6$

Exercise 1A (Revision)

1. How many integers are there between -2.5 and 3.5?
2. Find:
   (a) $-6 + -4$ **(b)** $-2 - -8$ **(c)** $-6 + 4$ **(d)** $-5 - 3$ **(e)** $7 - -3$
   (f) -5×-4 **(g)** 7×-3 **(h)** $-40 \div 5$ **(i)** $-42 \div -7$

1.3 Factors And Multiples

The **factors** of a number are all the whole numbers that divide into that number.

The **multiples** of a number are all the whole numbers that the number divides into.

Example 2

Write down the first five multiples of 6.

The **multiples** of 6 are the numbers in the 6 times table:
$1 \times 6 = 6$ $2 \times 6 = 12$ $3 \times 6 = 18$ $4 \times 6 = 24$ $5 \times 6 = 30$

So, the first five multiples of 6 are: 6, 12, 18, 24, 30, … and so on.

There are an infinite number of multiples.

Example 3

(a) Write down all the factors of 10.
(b) Write down the first 4 multiples of 10.

(a) The factors of 10 are the whole numbers that divide into 10.
The factors of 10 are 1, 2, 5 and 10.
(b) The first 4 multiples of 10 are 10, 20, 30, 40.

Note: Don't forget to include 10 itself! It is both a factor and a multiple of 10.

Fun fact: If you sometimes mix up the words **factor** and **multiple**, try this:
There are **few factors** (FF) and **many multiples** (MM).

A **common factor** is a whole number that is a factor of two or more numbers.

A **common multiple** is a whole number that is a multiple of two or more numbers.

Example 4

(a) Write down a common factor of 10 and 15.
(b) Is 36 a common multiple of 8 and 12?

(a) There are two possible answers: 1 and 5. These numbers are both common factors of 10 and 15.
(b) 36 is not a common multiple of 8 and 12. It is a multiple of 12, but not a multiple of 8.

Activity: Fizz Buzz

Play a game of 'Fizz Buzz' with a partner.

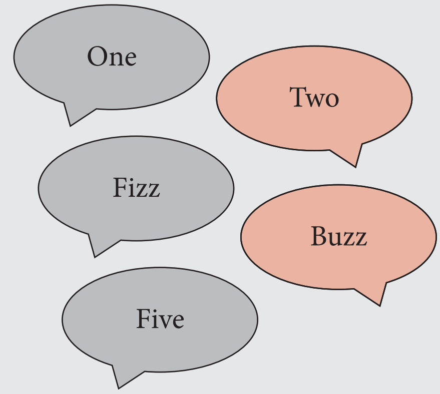
Take it in turns to count up from 1.

If you reach a multiple of 3 say 'fizz' instead of the number.

If you reach a multiple of 4 say 'buzz' instead of the number.

If the number is a multiple of 3 and of 4 then say 'fizz buzz'.

The first person to make a mistake loses the game!



Exercise 1B

1. Write down the first 5 multiples of:
   (a) 3 (b) 4 (c) 8 (d) 9
2. (a) List the first ten multiples of 3.
   (b) List the first five multiples of 6.
   (c) What is the relationship between the lists in part (a) and part (b)?
3. Write down:
   (a) all the multiples of 7 between 20 and 30,
   (b) all the multiples of 12 between 50 and 100,
   (c) all the multiples of 6 between 15 and 50.
4. Lily thinks of a number.
   She writes down some multiples of her number:

   6 24 48

   What number could Lily be thinking of? Find three different answers.
5. Find the answers to the following calculations.
   (a) The third multiple of 4 plus the second multiple of 10.
   (b) The fifth multiple of 2 multiplied by the second multiple of 6.
   (c) The sixth multiple of 4 minus the second multiple of 3.
   (d) The eighth multiple of 6 divided by the sixth multiple of 2.
6. Copy all the numbers below.

30	25	60	15	8	12
6	9	45	4	18	27
41	10	28	24	21	5

   (a) Draw a circle around each multiple of 3.
   (b) Draw a square around each multiple of 4.
   (c) Draw a triangle around each multiple of 5.
   (d) Which numbers need two shapes?
   (e) Which number needs 3 shapes?
7. Max and Grace play a game of 'Fizz Buzz'. The rules are described in the Activity above.
   They get up to 30 with no mistakes.
   (a) How many times did someone say 'fizz'? (Include 'fizz buzz'.)
   (b) How many times did someone say 'buzz'?
   (c) How many times did someone say 'fizz buzz'?

8. Charlie and Scott play a game of 'Fizz-Buzz-Bang'. The rules are the same as for 'Fizz Buzz' but now, if the number is a multiple of 5, they say 'bang'. What is the first number reached where someone says:
(a) 'fizz bang'? **(b)** 'buzz bang'? **(c)** 'fizz buzz bang'?

9. Copy and complete this multiplication grid.

×				
	6			
		15	25	
		21		49
	16		40	

10. Jamie thinks of a number. It is in the 4 times table and the 7 times table. Jamie's number is less than 100. What could Jamie's number be? How many possible answers are there?

11. Work out the lowest common multiple of:
(a) 2, 3 and 5. **(b)** 2, 3 and 6.

12. Reuben and Clare have a car race. Reuben's car completes the circuit in 7 seconds and Clare's car takes 5 seconds.
(a) How long is it before Clare's car overtakes Reuben's at the same time as they pass through the starting position?
(b) How many laps has each car completed at this time?

1.4 Prime Numbers

A **prime number** has only two factors: 1 and itself.

Example 5

(a) Is 7 a prime number?
(b) Is 9 a prime number?
(c) Is 11 a prime number?
(d) How many prime numbers are there between 20 and 30?

(a) 7 is a prime number. The only factors of 7 are 1 and 7.
(b) 9 is **not** a prime number because 3 is a factor of 9.
(c) 11 is a prime number. The only factors of 11 are 1 and 11.
(d) There are 2 prime numbers between 20 and 30: 23 and 29.

Activity: I'm Still Standing!

This is an ancient way to find prime numbers. It was devised by the Greek mathematician Eratosthenes who lived in the third century BC.

Step 1: The teacher gives everybody in the class a number from 2 to 30, or as far as possible. The whole class stands up.

Step 2: All those whose number is a multiple of 2, **except for the number 2 itself**, sit down.

Step 3: All those whose number is a multiple of 3, **except for the number 3 itself**, sit down.

Step 4: All those whose number is a multiple of 5, **except for the number 5 itself**, sit down.

Step 5 would be for those whose numbers are multiples of 7 (except for 7 itself) to sit down, and so on. Steps 5 and onwards are probably not necessary.

The people left standing should be the prime numbers!

Exercise 1C

1. Explain what is meant by a prime number. Give an example.
2. Which are the prime numbers in this list?
   1 5 12 19 30 31
3. Are there any even prime numbers? If so, what are they?
4. From the numbers in the box below, write down:
   (a) A prime number between 1 and 10.
   (b) A prime number between 10 and 20.
   (c) A prime number between 20 and 30.
   (d) A prime number bigger than 30.

110	13	26	75	12	7
95	37	9	27	29	93

5. Give a reason these numbers are not prime:
   (a) 28 (b) 21 (c) 77
6. Find two prime numbers that add up to make
   (a) 10 (b) 16 (c) 20
7. Are these numbers prime? If not, give a reason why not.
   (a) 63 (b) 200 (c) 685 (d) 243

1.5 Squares And Cubes

Squaring a number means multiplying it by itself.

A **square number** is another number squared. For example, 9 is a square number because $9 = 3^2$ or 3×3.

Cubing a number means multiplying it by itself and by itself again.

A **cube number** is another number cubed. For example, 8 is a cube number because $8 = 2^3$ or $2 \times 2 \times 2$.

Activity: I'm Still Standing! (New Rules)

Step 1: The teacher gives everybody in the class a number from 1 to 30, or as far as possible. Everybody starts standing up.

Step 2: All those whose number is a multiple of 2 sit down.

Step 3: All those whose number is a multiple of 3 change their position. Those who are standing sit down; those who are sitting stand up.

Step 4: All those whose number is a multiple of 4 change their position. Those who are standing sit down; those who are sitting stand up.

And so on, up to Step 30, or as far as possible. Who is left standing up?

Example 6

(a) Find (i) 5^2 and (ii) $(-5)^2$.
(b) Ethan says that 27 is a square number. Carly says it is a cube number. Who is right?
(c) Is 64 a square number or a cube number or both?

(a) (i) $5^2 = 5 \times 5$ (ii) $(-5)^2 = -5 \times -5 = 25$ (remember two negatives multiplied make a positive).
(b) Carly is right because $27 = 3^3$, so it is a cube number. It is not the square of another number.
(c) 64 is both a square number and a cube number:
- $64 = 8^2$, so it is a square number; and
- $64 = 4^3$, so it is a cube number.

Square roots and cube roots

Since $9 = 3^2$ then
- 9 is the square of 3; and
- 3 is the **square root** of 9.

Since $8 = 2^3$ then
- 8 is the cube of 2; and
- 2 is the **cube root** of 8.

Note: Be careful: most numbers have two square roots. In the last example we saw that 5^2 and $(-5)^2$ both give 25. This means that the two square roots of 25 are 5 and –5.

Example 7

(a) Bella says that the square of 81 is 9. Correct her statement.
(b) Sean says that the square root of 36 is 6. Correct his statement.

(a) Bella should say that the **square root** of 81 is 9, not the square.
(b) Sean should remember that 36 has two square roots: 6 and –6.

Exercise 1D

1. A square number is the square of another number. For example, 9 is a square number because it is 3^2. State whether these numbers are square numbers:
(a) 1 **(b)** 2 **(c)** 4 **(d)** 8 **(e)** 16 **(f)** 20

2. Look at the numbers in the box below.
(a) Which of the numbers in the box are:
  (i) square numbers? **(ii)** cube numbers?
(b) Which **two** of the numbers are neither a square number nor a cube number?
(c) Which **two** of the numbers are both a square number and a cube number?

1	4	8	9	16	24
25	27	36	40	49	64

3. Write down:
(a) The square roots of 64. **(b)** The cube root of 64.

4. Eithne says that 16 is the square of a square number. Is she right?

5. How many square numbers are there between 2 and 10?

6. **(a)** Andrea says that the square of 100 is 10. Correct her statement.
(b) Finn says that the square root of 25 is 5. Correct his statement.

1.6 Highest Common Factor And Lowest Common Multiple

The **highest common factor** (or **HCF**) of two numbers is the biggest number that is a factor of both.

The **lowest common multiple** (**LCM**) of two numbers is the smallest number that is a multiple of both.

Example 8

Find the lowest common multiple of 8 and 12.

We must find the **lowest** number that is a multiple of both 8 and 12. Write down the first few multiples of 8 and 12. Circle the numbers that appear in both lists.

Multiples of 8 are:	8	16	24	32	40	48	56	64	72
Multiples of 12 are:	12	24	36	48	60	72	84		

24, 48, 72, … are the **common multiples** of both 8 and 12.

So, 24 is the **lowest common multiple** of 8 and 12

Example 9

Find the highest common factor of 16 and 20.

We must find the **highest** number that is a factor of both 16 and 20. Write down all the factors of 16 and 20. Circle the numbers that appear in both lists.

Factors of 16 are:	1	2	4	8	16	
Factors of 20 are:	1	2	4	5	10	20

1, 2 and 4 are the **common factors** of 16 and 20.

So, 4 is the **highest common factor** of 16 and 20.

Some worded problems need to be answered using either highest common factors or lowest common multiples.

Example 10

In the Best Café, they make a great Ulster Fry. One chef can cook the bacon in 4 minutes. Another chef can cook the sausage in 6 minutes. Both chefs start cooking at 9:30 a.m. When are the bacon and sausage first ready at the same time?

This question requires the lowest common multiple of 4 and 6.

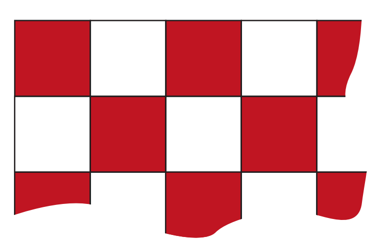
The bacon is ready after:	4	8	12	16	... minutes
The sausage is ready after:	6	12	18	24	... minutes

The smallest number in both lists is 12.

Bacon and sausage are both ready after 12 minutes. This is at 9:42 a.m.

Example 11

Jenni is sewing together squares of red and white material to make a check patterned cloth. The cloth must be 54 cm long and 36 cm wide. She doesn't want to cut any squares, so she must fit a whole number of squares along the length of the cloth and a whole number of squares along its width. The length and width of each square is to be a whole number of centimetres.



(a) Find the size of the largest square that Jenni could use.
(b) How many squares does Jenni use altogether?

(a) The size of each square must be a factor of 54 so that a whole number of them fit along the length. The size of each square must also be a factor of 36 so that a whole number of them fit along the width. So, the width is a common factor of 54 and 36. Since we are looking for the largest possible square, we must find the highest common factor.

Factors of 36 are: 1 2 3 4 6 9 12 18 36

Factors of 54 are: 1 2 3 6 9 18 27 54

The common factors have been circled. The highest common factor is 9.
So Jenni must use squares with a side length of 9 cm.

(b) If each square is to be 9 centimetres on each side, she needs 6 of them along the length and 4 of them along the width. In total she needs 24 squares.

Exercise 1E

1. **(a)** What are the first ten multiples of 4?
(b) What are the first ten multiples of 5?
(c) What is the lowest common multiple (LCM) of 4 and 5?

2. Find the lowest common multiple (LCM) of each of these pairs of numbers.
(a) 3 and 4 **(b)** 4 and 6 **(c)** 8 and 12

3. **(a)** Write down all the factors of 20.
(b) Write down all the factors of 24.
(c) What is the highest common factor (HCF) of 20 and 24?

4. Patrick says that the lowest common multiple of 10 and 15 is 150 as $10 \times 15 = 150$. Is Patrick right? Explain your answer fully.

5. Find:
(a) The lowest common multiple of 12 and 15.
(b) The highest common factor of 84 and 126.
(c) The highest common factor of 84 and 120.

6. $p = 28$ and $q = 36$
(a) Find the highest common factor of p and q.
(b) Find the lowest common multiple of p and q.

7. Trains leave Steel City heading towards Coketown every 8 minutes. Trains leave Steel City heading towards Coalville every 24 minutes. At 8:50 a.m. trains to both Coketown and Coalville leave Steel City. How many times do the two trains leave the station at the same time between 9 a.m. and 12 noon?

8. Conor and Aoife set the alarms on their phones to sound at 7:15 a.m. Both alarms sound together at 7:15 a.m. Conor's alarm then sounds every 8 minutes and Aoife's alarm sounds every 12 minutes. At what time do the two alarms next sound together?

9. Gunk shampoo comes in two different size bottles: one containing 300 ml, the other containing 380 ml. Jenny works in the Gunk Shop. She needs to order two large tubs of Gunk shampoo. One tub is used to refill 300 ml bottles a whole number of times; the other tub will be used to refill 380 ml bottles a whole number of times. Given that the two tubs are the same size, find the smallest possible size of the tubs.

10. A piece of string 132 cm long is to be cut into equal pieces. A second piece of string 48 cm long is to be cut into the same size pieces. If the pieces are a whole number of centimetres long, what is the largest possible length of these pieces?

1.7 Index Notation

You should understand index notation.

Example 12

Without a calculator, find the value of:

(a) 5^2 **(b)** 4^3 **(c)** 10^4

Using a calculator, find the value of:

(d) 2^{10} **(d)** $9^3 + 10^3$

(a) $5^2 = 5 \times 5 = 25$

(b) $4^3 = 4 \times 4 \times 4 = 64$

(c) $10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$

(d) From the calculator, $2^{10} = 1024$

(e) $9^3 + 10^3 = 729 + 1000 = 1729$

Note: Fun fact: you might assume that a kilobyte of data is 1000 bytes, but unusually it is defined as being 2^{10} or 1024 bytes.

You should know how to say these calculations. From Example 7 above:

- $5^2 = 25$ (we say '5 squared equals 25')
- $4^3 = 64$ (we say '4 cubed equals 64')
- $2^{10} = 1024$ (we say '2 to the power of 10 equals 1024')

1.8 Prime Factorisation

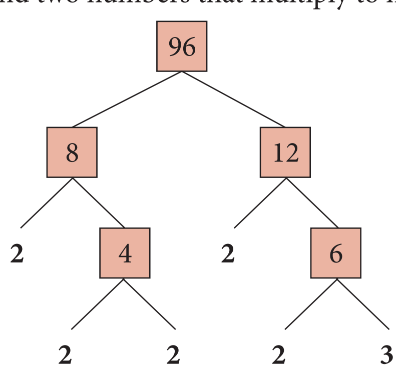
Every integer can be written as a **product of prime factors**. The **product of prime factors** is also called the **prime factorisation** for that integer.

Example 13

Write 96 as a product of prime factors.

Method 1: Factor trees

Find two numbers that multiply to make 96. The numbers 8 and 12 have been chosen.



Repeat the process for 8 and for 12. Repeat until you are only left with prime numbers. These primes make up the product of prime factors.

So: $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

or, using index notation: $96 = 2^5 \times 3$

Method 2: Repeated division

Choose the smallest prime number that is a factor of 96, which is 2. Divide 96 by 2, giving 48. Repeat the process until you reach 1.

$$\begin{array}{r|l} 2 & 96 \\ 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

The prime factorisation for 96 is the product of the prime numbers you divided by.

So: $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

or, using index notation: $96 = 2^5 \times 3$

Note: For any integer, there is only one way to write it using a product of prime factors.

Example 14

The prime factorisation for 510 is $2 \times 3 \times 5 \times 17$. Are the following numbers factors of 510?

(a) 2 **(b)** 3 **(c)** 6 **(d)** 11 **(e)** 25 **(f)** 30

(a) Yes, because 2 is a part of the prime factorisation.
(b) Yes, because 3 is a part of the prime factorisation.
(c) Yes, because $6 = 2 \times 3$
(d) No, because 11 is a prime that does not appear in the prime factorisation.
(e) No, because $25 = 5^2$. There is only one 5 in the prime factorisation.
(f) Yes, because $30 = 2 \times 3 \times 5$

Exercise 1F

1. Write as a product of prime factors:
(a) 108 **(b)** 120

2. Match each number to its prime factorisation.

120	990	240	7425
$3^3 \times 5^2 \times 11$	$2 \times 3^2 \times 5 \times 11$	$2^4 \times 3 \times 5$	$2^3 \times 3 \times 5$

3. Find the prime factorisation for each of these numbers, giving your answers in index form.
(a) 45 **(b)** 150 **(c)** 48 **(d)** 126 **(e)** 243

4. The prime factorisation for 462 is $2 \times 3 \times 7 \times 11$. Using this, determine whether the following numbers are factors of 462.
(a) 2 **(b)** 3 **(c)** 6 **(d)** 7 **(e)** 13

5. The prime factorisation for 875 is $5^3 \times 7$. Using this, determine whether the following numbers are factors of 875.
(a) 3 **(b)** 5 **(c)** 7 **(d)** 25 **(e)** 11 **(f)** 49

6. Write these as a product of prime factors.
(a) $6^2 \times 10$ **(b)** $5^2 \times 10$ **(c)** $6^3 \times 10^2$

1.9 Summary

In this chapter you have learnt about several special types of whole number.

You revised working with **negative numbers**.

You have also learnt that:

- A **factor** of a number is a number that divides into it. For example, 3 is a factor of 12.
- A **multiple** of a number is in the times table of that number. For example, 15 is a multiple of 5.
- A **prime number** has no factors apart from 1 and itself. For example, 7 is a prime number because its only factors are 1 and 7.
- A **square number** is another number multiplied by itself. For example, 9 is a square number because it is 3×3 or 3^2.
- A **cube number** is another number multiplied by itself and by itself again. For example, 8 is a cube number because it is $2 \times 2 \times 2$ or 2^3.
- A **square root** is the number that multiplies by itself to give a number. For example, the square root of 9 is 3. We write $\sqrt{9} = 3$.
- A **cube root** is the number that multiplies by itself and by itself again to give a number. For example, the cube root of 8 is 2. We write $\sqrt[3]{8} = 2$.
- A **common factor** is a factor of two or more numbers. For example, 2 is a common factor of 8 and 12.
- The **highest common factor (HCF)** of two or more numbers is the largest number that is a common factor of two or more numbers. For example 4 is the highest common factor of 8 and 12.
- A **common multiple** is a multiple of two or more numbers. For example, 40 is a common multiple of 4 and 10.
- The **lowest common multiple (LCM)** of two or more numbers is the smallest number that is a common multiple of two or more numbers. For example, 20 is the lowest common multiple of 4 and 10.

Chapter 11
Coordinate Geometry

11.1 Introduction

This chapter is about straight lines plotted on the coordinate grid. The equation of a straight line is written in the form $y = mx + c$, where m represents the slope of the line and c represents the y-intercept. The slope of a line is the measure of the amount that the line rises or falls for each unit it moves to the right. The y-intercept is the point at which the line crosses the y-axis.

Key words

- **Gradient**: The gradient is a measure of the steepness.
- ***x*-intercept**: Where a line passes through the x-axis.
- ***y*-intercept**: Where a line passes through the y-axis.
- **Line segment**: A part of a straight line between two points on a graph.
- **Midpoint**: The point at the centre of a line segment.

Before you start you should know how to:

- Plot points on the coordinate grid in all four quadrants.
- Read the coordinates of points plotted on the coordinate grid in all four quadrants.

In this chapter you will learn how to:

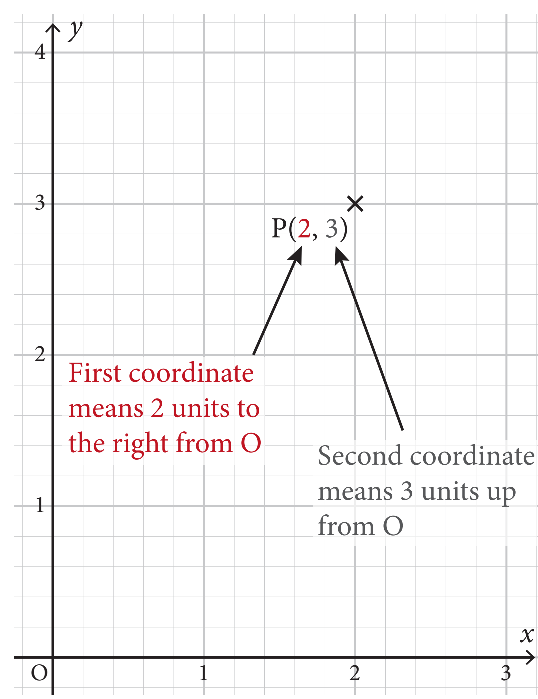
- Draw a straight line from a list of points.
- Take readings from a straight-line graph.
- Find the midpoint of a line segment.
- Find the length of a line segment.
- Find the gradient and intercepts of a line.

11.2 Revision of Coordinates

To draw any feature on a map we need two values – a latitude and a longitude – to fully specify its position. Coordinates use two values to specify the location of a point:

- The first value, called the x-coordinate, specifies how far in the direction along the horizontal x-axis the point is.
- The second value, called the y-coordinate, specifies how far in the direction of the vertical y-axis the point is.

A coordinate pair is written between round brackets with a comma between. All the distances are measured from the point where the horizontal; (x-) and vertical (y-) axes intersect. This starting point is called the origin and referred to as O or (0, 0).



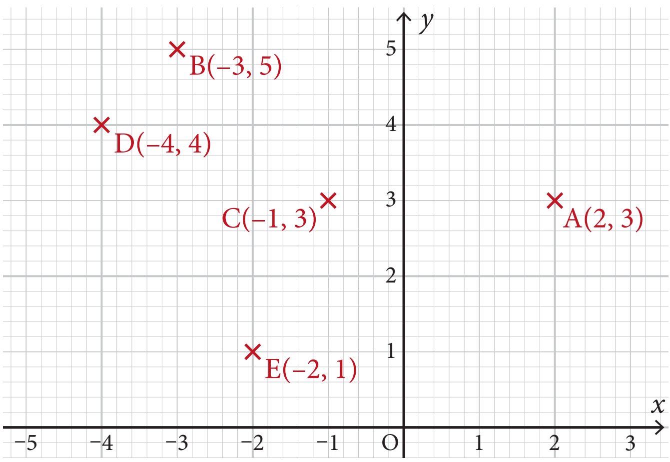
So to plot the point P that is 2 units to the right of the origin and 3 units above the origin we would write P(2, 3) and plot as shown in the diagram on the previous page.

Note: Every point on a graph has a pair of coordinates in the following form:
(number along the horizontal axis , number up the vertical axis)

Points that lie to the left of the origin O have negative *x*-coordinates.

Example 1

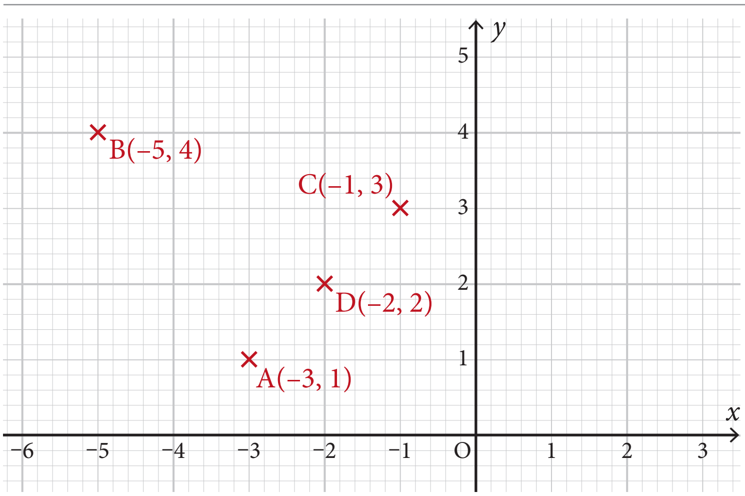
Plot the points A(2, 3), B(−3, 5), C(−1, 3), D(−4, 4) and E(−2, 1) on a coordinate grid.



For points in the top left quadrant, only the *x*-coordinate is negative.

Example 2

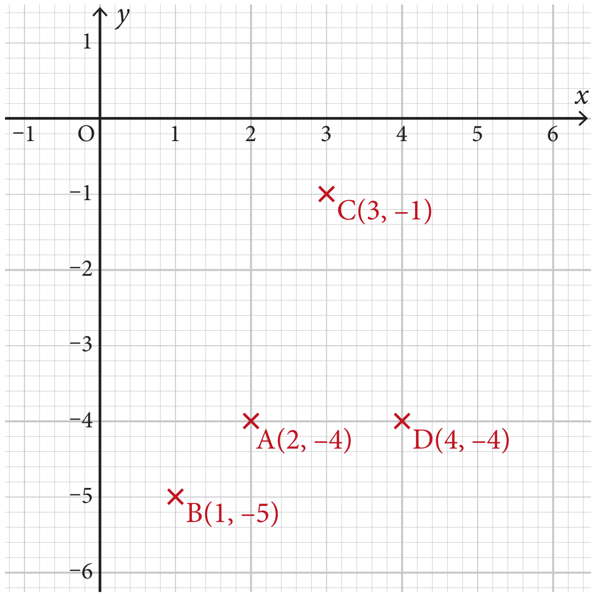
Plot the points A(−3, 1), B(−5, 4), C(−1, 3) and D(−2, 2).



In the same way, points below the level of the origin have negative *y*-coordinates. Plotting in the bottom right quadrant, only the *y*-coordinate is negative.

Example 3

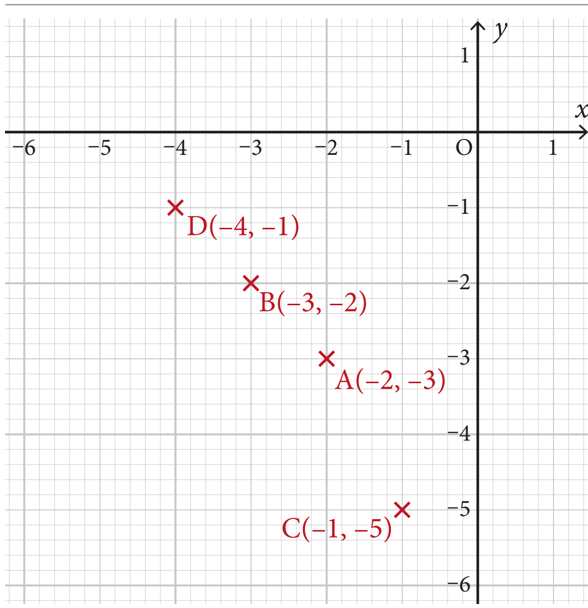
Plot the points A(2, –4), B(1, –5), C(3, –1) and D(4, –4).



Plotting in the bottom left quadrant, both coordinates are negative.

Example 4

Plot the points A(–2, –3), B(–3, –2), C(–1, –5) and D(–4, –1).



Exercise 11A (Revision)

1. (a) Draw coordinate axes with values ranging from –8 to 5 on the x-axis and –8 to 8 on the y-axis.
   (b) Plot the following points and join each point to the next one with a straight line.
   (–1, –7), (1, –7), (2, –5), (2, –4.5), (3, –3.5), (3, –1), (4.5, 1), (5, 2), (4, 2), (2.5, 4), (1, 6.5), (0, 6), (–2, 6), (–2.5, 7), (–5, 7), (–7, 6), (–8, 5), (–8, 3), (–7, 1), (–6, 0.5), (–5, 1), (–3, 1), (–3, 0), (–2, –2), (–2.5, –3), (–2, –4), (–1, –7)
   (c) What continent have you drawn?

11.3 Drawing and Taking Readings From Straight-Line Graphs

Drawing a straight-line graph from a list of points

The first skill you need is drawing a straight line if you are given a list of points on it. This list could be a simple list of coordinate points.

Example 5

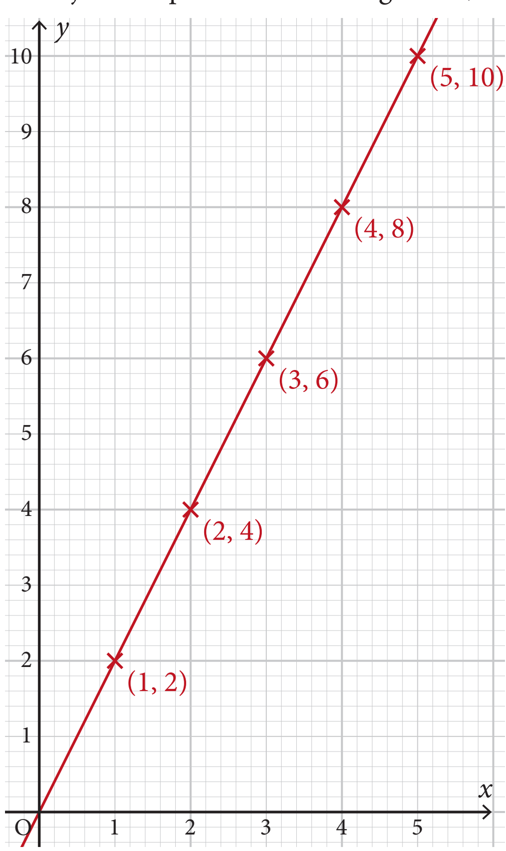
Plot the straight line passing through the points (1, 2), (2, 4), (3, 6), (4, 8) and (5, 10)

Note: this information may sometimes be given in the form of a table:

x	1	2	3	4	5
y	2	4	6	8	10

Plot each point.

Then join the points with a straight line, extending the line in both directions.



Note: The points of every straight line are calculated from a formula. In this case, the formula is $y = 2x$

Drawing a straight-line graph from the equation of the line

If you are given the equation of the line, you can use it to plot points on the line. You may be asked to complete a table of values, as in the next example. Using the x-values you are given, use the equation to find the corresponding y-values. Then, you can plot these points and draw a line through them.

Example 6

(a) Complete the table of values for the line $y = x - 2$

(b) Plot the line.

x	0	1	2	3	4	5
y	–2	–1		1		3

(a) To find the y-value corresponding to $x = 2$, substitute $x = 2$ into the equation of the line $y = x - 2$

So $y = 2 - 2 = 0$ When $x = 2$, $y = 0$

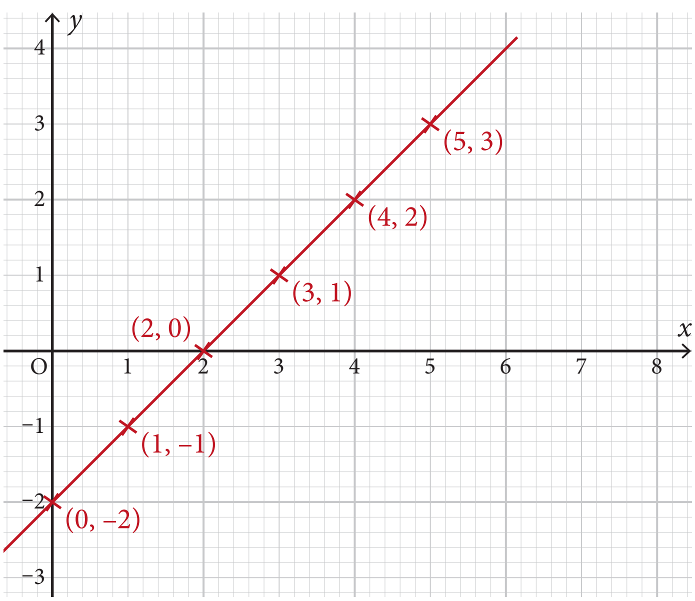
To find the y-value corresponding to $x = 4$, substitute $x = 4$ into the equation of the line $y = x - 2$

So $y = 4 - 2 = 2$ When $x = 4$, $y = 2$

Our completed table looks like:

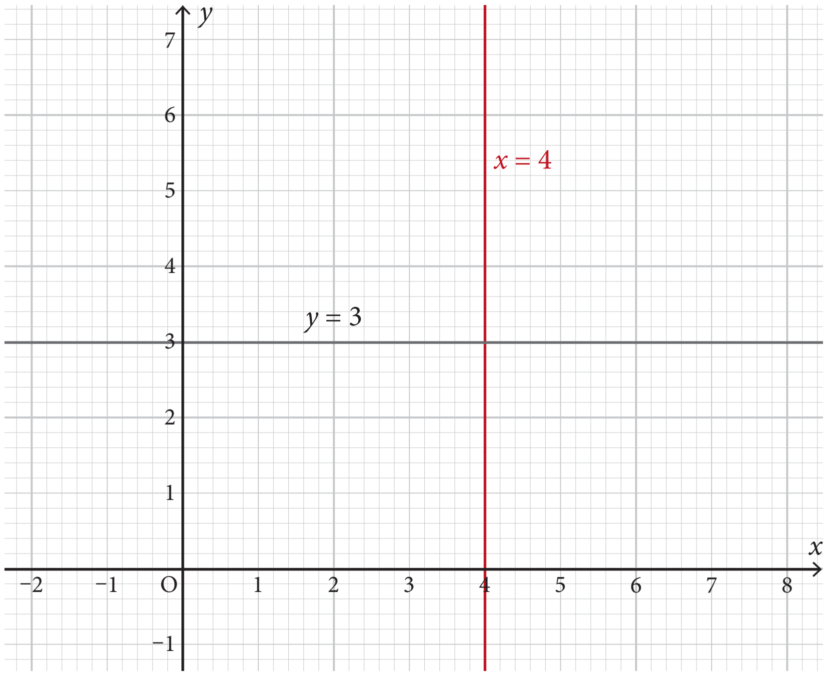
x	0	1	2	3	4	5
y	–2	–1	0	1	2	3

(b) Plotting the line gives:



Horizontal and vertical lines

Some equations produce horizontal and vertical lines when drawn on a graph. The line $y = 3$ is a horizontal line passing through 3 on the y-axis. The line $x = 4$ is a vertical line passing through 4 on the x-axis. The following graph shows these two lines.



The line $y = 3$ has this equation because every point on the line has a y-coordinate of 3

The line $x = 4$ has this equation because every point on the line has an x-coordinate of 4

Taking readings from a straight-line graph

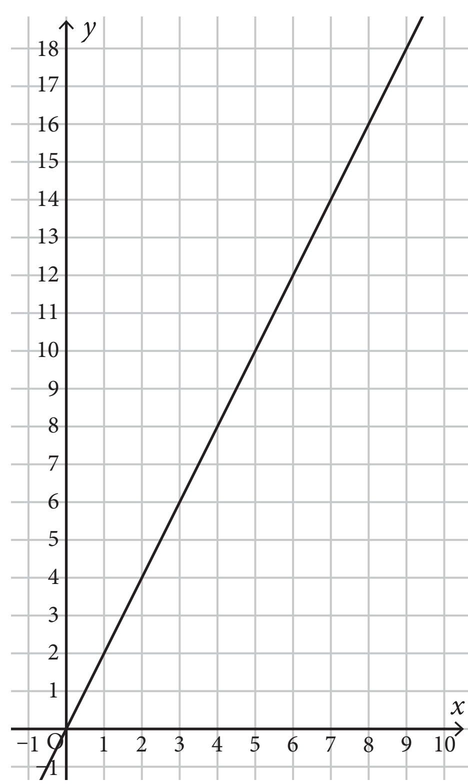
The next skill is taking readings from a straight-line graph. The graph may be given to you without the equation of the line and you may be asked to take readings from it, as in the next example.

Example 7

Use the graph on the right to:

(a) Find the y-value corresponding to $x = 8$

(b) Find the x-value when $y = 6$

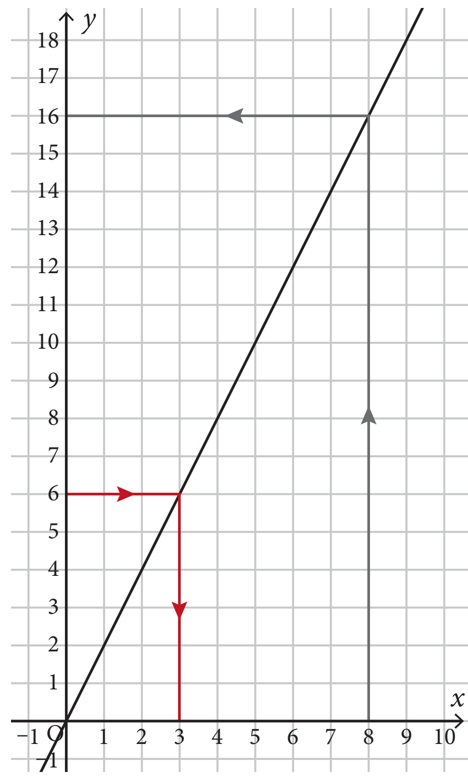


(a) Draw a vertical line (shown in grey) from $x = 8$ on the x-axis up to the graph. Then draw a horizontal line from the graph to the y-axis to find the corresponding y-value.

When $x = 8$, the y-value is 16

(b) Draw a horizontal line (shown in red) from 6 on the y-axis to the graph. Then draw a vertical line from the graph to the x-axis.

When $y = 6$, the x-value is 3



Exercise 11B

1. Draw the graph of the line passing through the points given in the table below.

x	0	1	2	3	4	5	6	7
y	–1	1	3	5	7	9	11	13

2. Draw the graph of the line passing through the points given in the table below.

x	1	2	3	4	5	6
y	3	4	5	6	7	8

3. Complete the table below and hence draw the graph of $y = 2x + 1$

x	–2	–1	0	1	2	3	4
y	–3	–1	1		5		9

4. Complete the table below and hence draw the graph of $y = 3x – 1$

x	0	1	2	3	4	5	6
y	–1		5	8		14	

5. Complete the table below and hence draw the graph of $y = 3x + 1$

x	–2	–1	0	1	2	3	4
y	–5	–2	1		7		13

6. Complete the table below and hence draw the graph of $y = x – 3$

x	0	1	2	3	4	5	6
y	–3		–1	0		2	

7. Complete the table below and hence draw the graph of $y = 2 + 3x$

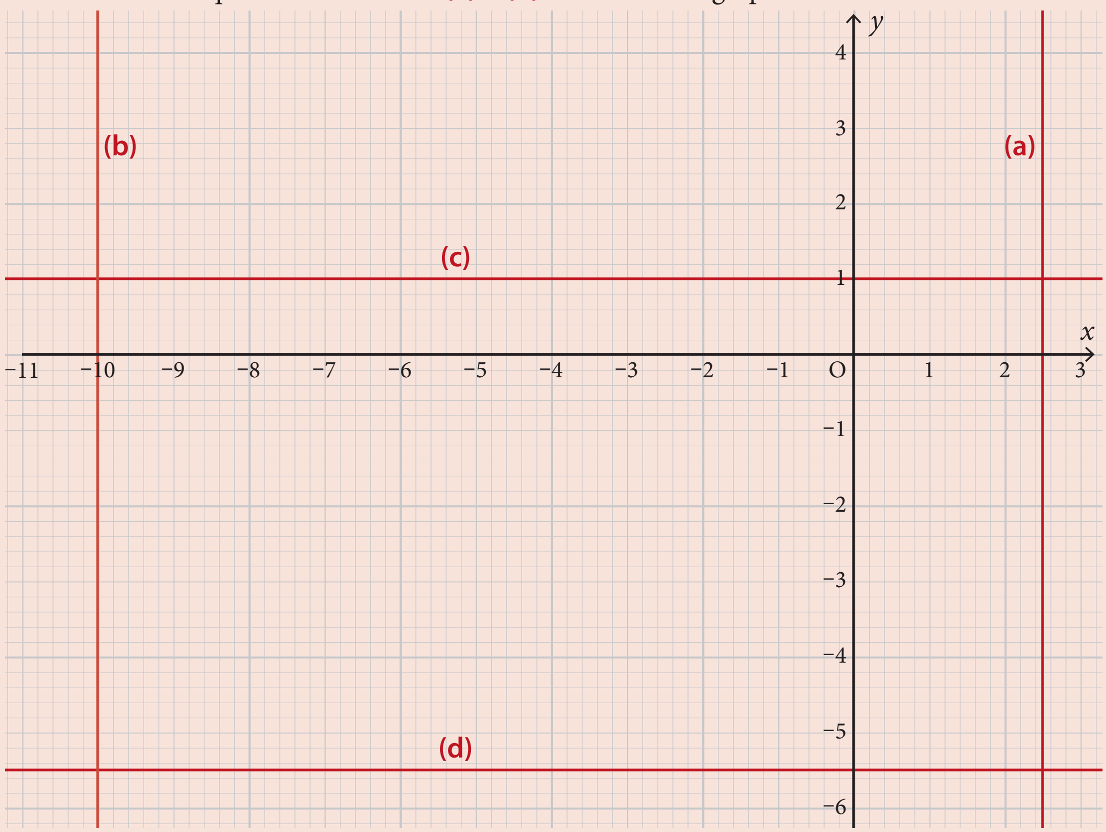
x	–1	0	1	2	3	4	5
y	–1	2	5		11		17

8. Complete the table below and hence draw the graph of $y = 2 - x$

x	0	1	2	3	4	5	6
y	2	1	0		–2	–3	

9. On the same graph, draw these lines:
(a) $x = 3$ **(b)** $y = 1$ **(c)** $x = -5$ **(d)** $y = -3.5$

10. Write down the equation of each line **(a)** – **(d)** shown on the graph below.



11. Look at the graph on the right. The equation of the straight line is $y = \frac{1}{2}x + 1$

(a) Use the graph to find the y-value corresponding to an x-value of 8

(b) Use the graph to find the x-value corresponding to a y-value of 6