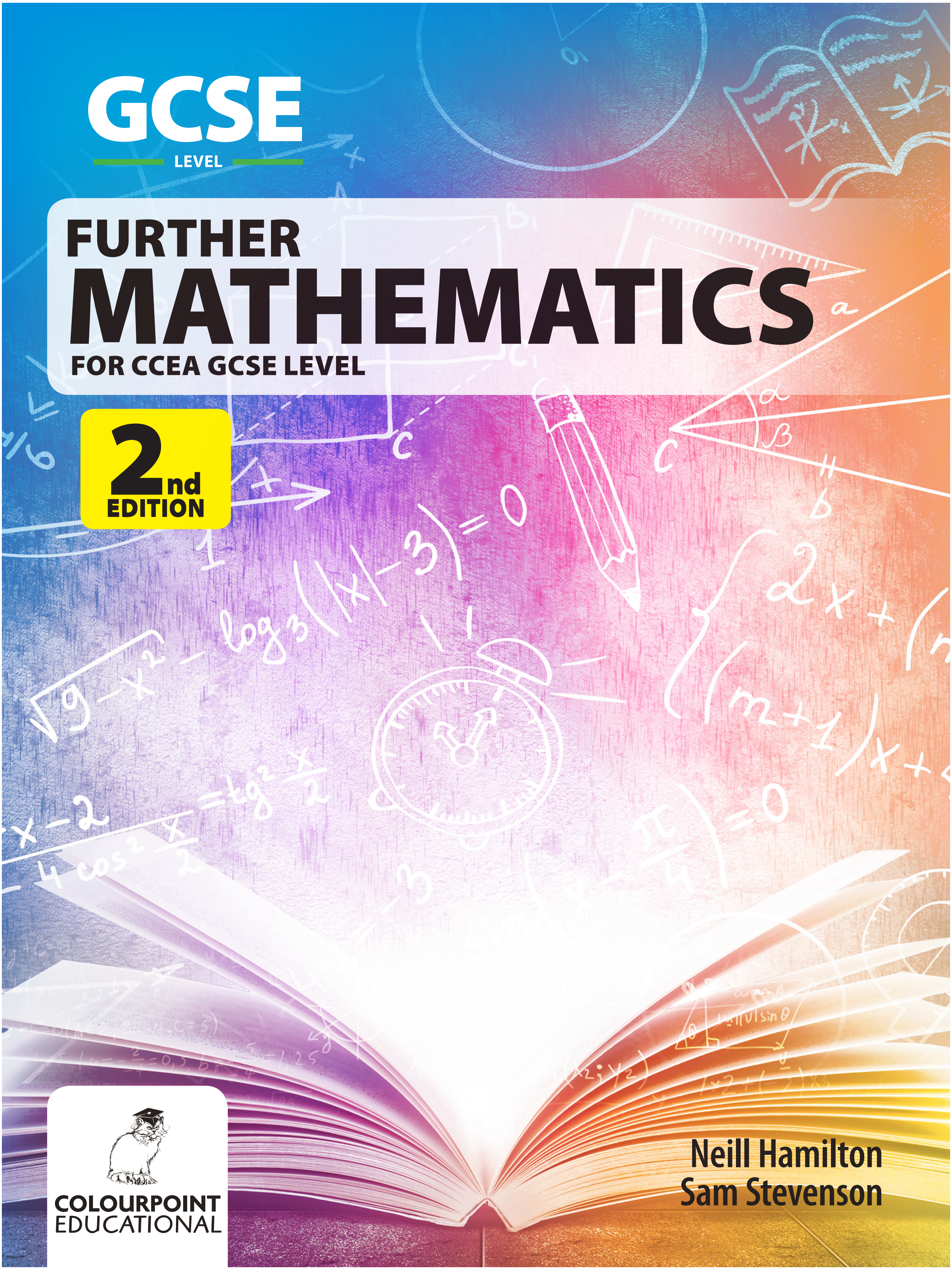

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LEVEL

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Neill Hamilton
Sam Stevenson

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Colourpoint House
Jubilee Business Park
21 Jubilee Road
Newtownards
County Down
Northern Ireland
BT23 4YH

Tel: 028 9182 6339
Fax: 028 9182 1900
E-mail: sales@colourpoint.co.uk
Web site: www.colourpointeducational.com

The Authors

Neill Hamilton will be well known to Mathematics teachers in Northern Ireland. Until his retirement in 2012, he was a teacher of GCSE Mathematics and Additional/Further Mathematics at a Northern Ireland comprehensive school. His previous publications include GCSE Maths Revision Booklets T3 and T4, also published by Colourpoint.

Sam Stevenson has taught Mathematics and Informatics in grammar schools to a generation. He has a passion for developing and delivering Further Mathematics at both GCSE and GCE levels. He will be known to many from the Northern Ireland Mathematics educational circuit. His hobbies include family, cycling, keeping tortoises, skiing and combinatorics, in no particular order.

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Unit 1: Pure Mathematics

CHAPTER 1: SIMPLIFYING ALGEBRAIC EXPRESSIONS

1.1 Adding And Subtracting Algebraic Expressions

When adding or subtracting you need to find the lowest common denominator. For example:

1. Simplify: $\dfrac{2x+3}{5} + \dfrac{2x-3}{6}$

   The lowest common denominator of 5 and 6 is 30. So we change each fraction to a denominator of 30.

   This gives: $\dfrac{12x+18}{30} + \dfrac{10x-15}{30}$

   We can then add these fractions to get: $\dfrac{22x+3}{30}$

2. Simplify: $\dfrac{2x-5}{6} - \dfrac{4-2x}{3}$

   The lowest common denominator of 6 and 3 is 6. So we change the second fraction to a denominator of 6.

   This gives: $\dfrac{2x-5}{6} - \dfrac{8-4x}{6}$

   We can then subtract these fractions to get:

   $\dfrac{6x-13}{6}$

3. Simplify: $\dfrac{3}{5x-2} + \dfrac{2}{1-2x}$

   The lowest common denominator is $(5x-2)(1-2x)$. So we change each fraction to a denominator of $(5x-2)(1-2x)$.

   This gives: $\dfrac{3-6x}{(5x-2)(1-2x)} + \dfrac{10x-4}{(5x-2)(1-2x)}$

   We can then add these fractions to get:

   $\dfrac{4x-1}{(5x-2)(1-2x)}$

   You do not need to work out the brackets on the denominator.

Exercise 1A: *Simplify the following.*

1. $\dfrac{x+7}{4} + \dfrac{2x-1}{3}$
2. $\dfrac{4x-3}{5} - \dfrac{x+2}{2}$
3. $\dfrac{3x-2}{6} + \dfrac{1-2x}{4}$
4. $\dfrac{2x-5}{4} - \dfrac{2-x}{8}$
5. $\dfrac{3x-2}{4} - \dfrac{2x-1}{10}$
6. $\dfrac{4}{3x-2} - \dfrac{2}{x+5}$
7. $\dfrac{5}{2-x} + \dfrac{2}{x+4}$
8. $\dfrac{4}{3-2x} - \dfrac{2}{x-3}$
9. $\dfrac{5}{2x-1} + \dfrac{3}{x-4}$
10. $\dfrac{5}{x-3} + \dfrac{2}{2x+1}$

1.2 Adding And Subtracting Algebraic Fractions Where Each Denominator Has A Product of Two Terms

For example:

4. Simplify: $\dfrac{6}{(x+5)(2x-3)} + \dfrac{2}{x(2x-3)}$

   The lowest common denominator is $x(x+5)(2x-3)$. So we change each fraction to a denominator of $x(x+5)(2x-3)$.

   This gives: $\dfrac{6x}{x(x+5)(2x-3)} + \dfrac{2x+10}{x(x+5)(2x-3)}$

   We can then add these fractions to get:

   $\dfrac{8x+10}{x(x+5)(2x-3)}$

   You do not need to work out the brackets on the denominator.

5. Simplify: $\dfrac{5}{(x-6)(3x+2)} - \dfrac{3}{(x+4)(x-6)}$

   The lowest common denominator is $(x+4)(x-6)(3x+2)$. So we change each fraction to a denominator of $(x+4)(x-6)(3x+2)$.

This gives:

$$\frac{5x+20}{(x+4)(x-6)(3x+2)} - \frac{9x+6}{(x+4)(x-6)(3x+2)}$$

We can then subtract these fractions to get:

$$\frac{-4x+14}{(x+4)(x-6)(3x+2)}$$

Exercise 1B: *Simplify the following.*

1. $\frac{7}{(x+2)(x-3)} + \frac{4}{x(x-3)}$
2. $\frac{5}{2x(x-3)} + \frac{2}{(x+2)(x-3)}$
3. $\frac{4}{(x+5)(x+3)} + \frac{3}{(x-1)(x+5)}$
4. $\frac{5}{(x-2)(2x+1)} + \frac{1}{(x-2)(x+4)}$
5. $\frac{3}{x(x-4)} + \frac{2}{x(x+2)}$
6. $\frac{4}{(x-2)(x+1)} - \frac{3}{(x+1)(x+5)}$
7. $\frac{5}{2x(x-3)} - \frac{2}{(x+4)(x-3)}$
8. $\frac{3}{(x-1)(x+4)} - \frac{4}{(x+4)(x-2)}$
9. $\frac{2}{(x+5)(x-2)} - \frac{3}{(x+5)(x+2)}$
10. $\frac{3}{(x+2)(2x-1)} - \frac{2}{(x-4)(2x-1)}$

1.3 Adding And Subtracting Algebraic Fractions With Quadratic Denominators

First, factorise the denominators. Then add or subtract as before. For example:

6. Simplify: $\frac{4}{x^2-3x} + \frac{2}{x^2-x-6}$

$x^2-3x = x(x-3)$ by common factors and
$x^2-x-6 = (x+2)(x-3)$ by quadratic factors.
So we can write the expression as:

$$\frac{4}{x(x-3)} + \frac{2}{(x+2)(x-3)}$$

The lowest common denominator is $x(x+2)(x-3)$. So we change each fraction to a denominator of $x(x+2)(x-3)$.

This gives: $\frac{4x+8}{x(x+2)(x-3)} + \frac{2x}{x(x+2)(x-3)}$

We can then add these fractions to get:

$$\frac{6x+8}{x(x+2)(x-3)}$$

7. Simplify: $\frac{5}{3x^2+10x-8} - \frac{2}{x^2-16}$

$3x^2+10x-8 = (3x-2)(x+4)$ by quadratic factors and $x^2-16 = (x-4)(x+4)$ by difference of two squares. So we can write the expression as:

$$\frac{5}{(3x-2)(x+4)} - \frac{2}{(x-4)(x+4)}$$

The lowest common denominator is $(x-4)(x+4)(3x-2)$.

So we change each fraction to a denominator of $(x-4)(x+4)(3x-2)$. This gives:

$$\frac{5x-20}{(x-4)(x+4)(3x-2)} - \frac{6x-4}{(x-4)(x+4)(3x-2)}$$

We can then subtract these fractions to get:

$$\frac{-x-16}{(x-4)(x+4)(3x-2)}$$

Exercise 1C: *Simplify the following.*

1. $\frac{4}{3x^2+6x} + \frac{3}{x^2+3x+2}$
2. $\frac{5}{x^2+2x-8} + \frac{2}{x^2-4}$
3. $\frac{3}{2x^2+7x-4} + \frac{5}{x^2+x-12}$
4. $\frac{3}{2x^2-10x} + \frac{2}{2x^2-11x+5}$
5. $\frac{4}{x^2+3x-10} + \frac{3}{x^2-7x+10}$
6. $\frac{4}{2x^2-x} - \frac{2}{x^2+x}$

Exercise 1C...

7. $\frac{3}{x^2 - 2x - 8} - \frac{2}{x^2 - 16}$

8. $\frac{2}{2x^2 - x - 6} - \frac{3}{x^2 - 2x}$

9. $\frac{4}{x^2 + x - 12} - \frac{5}{2x^2 + 13x + 20}$

10. $\frac{3}{4x^2 + 7x - 2} - \frac{2}{x^2 - x - 6}$

1.4 Multiplying And Dividing Algebraic Expressions

Multiplying

Cancel as far as possible. Then multiply the numerators and then multiply the denominators. For example:

8. $\frac{6x^3}{y^4t^2} \times \frac{10y^2}{8tx}$

You can divide 2 into 6 and 8 to get 3 and 4.

You can then divide 2 into 10 and 4 to get 5 and 2.

You can cancel x^3 and x by x to get to x^2 and 1.

You can cancel y^2 and y^4 by y^2 to get to 1 and y^2.

This gives: $\frac{3x^2}{y^2t^2} \times \frac{5}{2t}$

You can then multiply these to get the answer: $\frac{15x^2}{2y^2t^3}$

Dividing

Dividing by a fraction is the same as multiplying by its reciprocal. So, for example:

9. $\frac{10a^3b}{6b^2} \div \frac{4c}{5a^2}$

The reciprocal of $\frac{4c}{5a^2}$ is $\frac{5a^2}{4c}$

So we can multiply: $\frac{10a^3b}{6b^2} \times \frac{5a^2}{4c}$

You can divide 2 into 10 and 4 to get 5 and 2.

You can cancel b and b^2 by b to get to 1 and b.

This gives $\frac{5a^3}{6b} \times \frac{5a^2}{2c}$

You can then multiply these to get the answer: $\frac{25a^5}{12bc}$

Exercise 1D: *Simplify the following.*

1. $\frac{5x^2}{yv} \times \frac{2y^2}{vx}$
2. $\frac{3vt}{2n^2} \times \frac{4n}{6v^2}$
3. $\frac{10q}{r^2} \times \frac{2qr}{5n}$
4. $\frac{4vw}{2t} \times \frac{9t^2}{2v^3}$
5. $\frac{6qr}{2v^2} \times \frac{10vw}{3q^2}$
6. $\frac{9a^2}{5b} \div \frac{3c}{2ab^3}$
7. $\frac{4q^2}{3r} \div \frac{5q}{6rn}$
8. $\frac{3t^2}{2vw} \div \frac{9tn}{4v^2}$
9. $\frac{9a}{2b^2} \div \frac{7a^2}{4bc}$
10. $\frac{4v^2}{3q} \div \frac{2r}{5vq^2}$

1.5 Dividing Algebraic Fractions With Quadratic Numerators And Denominators

First, factorise each quadratic expression. Then cancel as far as possible. For example:

10. Simplify: $\frac{14x^2 - 7x}{4x^2 - 1}$

$14x^2 - 7x = 7x(2x - 1)$ by common factors and

$4x^2 - 1 = (2x - 1)(2x + 1)$ by difference of two squares.

So you can rewrite the fraction as $\frac{7x(2x - 1)}{(2x - 1)(2x + 1)}$

You can then cancel out the $(2x - 1)$ to get the final answer:

$\frac{7x}{2x + 1}$

11. Simplify: $\frac{x^2 + 2x - 15}{3x^2 + 13x - 10}$

$x^2 + 2x - 15 = (x - 3)(x + 5)$ by quadratic factors and

$3x^2 + 13x - 10 = (3x - 2)(x + 5)$ by quadratic factors.

So you can rewrite the fraction as $\frac{(x - 3)(x + 5)}{(3x - 2)(x + 5)}$

You can then cancel out the $(x + 5)$ to get the final answer:

$\frac{x - 3}{3x - 2}$

Exercise 1E: *Simplify the following.*

1. $\dfrac{2x^2 + 5x - 3}{x^2 + x - 6}$
2. $\dfrac{3x^2 - 6x}{2x^2 - 5x + 2}$
3. $\dfrac{x^2 - 25}{x^2 - x - 20}$
4. $\dfrac{4x^2 + 5x - 6}{x^2 - x - 6}$
5. $\dfrac{2x^2 + 9x - 5}{2x^2 + 3x - 2}$
6. $\dfrac{4x^2 - 12x}{x^2 - x - 6}$
7. $\dfrac{2x^2 + 7x - 4}{x^2 + 2x - 8}$
8. $\dfrac{3x^2 + 13x - 10}{6x^2 + 5x - 6}$
9. $\dfrac{8x^2 - 4x}{4x^2 - 1}$
10. $\dfrac{5x^2 + 13x - 6}{x^2 - 2x - 15}$

1.6 Multiplying And Dividing Quadratic Numerators And Denominators

First, factorise and cancel as far as possible. Then multiply or divide. For example:

12. $\dfrac{2q^2 - 7q + 3}{4} \times \dfrac{6}{q^2 - 9}$

First you need to factorise each quadratic expression:

$2q^2 - 7q + 3 = (q - 3)(2q - 1)$ by quadratic factors and

$q^2 - 9 = (q - 3)(q + 3)$ by difference of two squares.

So you can rewrite the multiplication as:

$$\frac{(q - 3)(2q - 1)}{4} \times \frac{6}{(q - 3)(q + 3)}$$

You can divide 6 and 4 by 2 to get 3 and 2. You can then cancel out the $(q - 3)$ to get the final answer:

$$\frac{3(2q - 1)}{2(q + 3)}$$

13. $\dfrac{x^2 + 4x}{5} \div \dfrac{x^2 + 2x - 8}{10}$

The reciprocal of $\dfrac{x^2 + 2x - 8}{10}$ is $\dfrac{10}{x^2 + 2x - 8}$

So we can multiply: $\dfrac{x^2 + 4x}{5} \times \dfrac{10}{x^2 + 2x - 8}$

You need to factorise each quadratic expression.

$x^2 + 4x = x(x + 4)$ by common factors and

$x^2 + 2x - 8 = (x + 4)(x - 2)$ by quadratic factors.

So you can rewrite the multiplication as:

$$\frac{x(x + 4)}{5} \times \frac{10}{(x + 4)(x - 2)}$$

You can divide 10 and 5 by 5 to get 2 and 1. You can then cancel out the $(x + 4)$ to get the final answer:

$$\frac{2x}{x - 2}$$

Exercise 1F: *Work out the following.*

1. $\dfrac{x^2 - 2x}{3} \times \dfrac{6}{x^2 + 2x - 8}$
2. $\dfrac{2x^2 - 5x - 3}{4} \times \dfrac{10}{6x^2 - x - 2}$
3. $\dfrac{x^2 + 2x - 15}{8} \times \dfrac{6}{x^2 - 5x + 6}$
4. $\dfrac{3x^2 + 14x + 8}{6} \times \dfrac{9}{x^2 + 2x - 8}$
5. $\dfrac{4x^2 - 10x}{10} \times \dfrac{8}{3x^2 + 12x}$
6. $\dfrac{4x^2 + 8x}{3} \div \dfrac{3x^2 - x}{6}$
7. $\dfrac{x^2 - 9x + 14}{4} \div \dfrac{x^2 + 3x - 10}{2}$
8. $\dfrac{2x^2 + 9x - 5}{8} \div \dfrac{8x^2 + 2x - 3}{6}$
9. $\dfrac{x^2 + 9x + 18}{9} \div \dfrac{x^2 + x - 6}{12}$
10. $\dfrac{4x^2 + 7x - 2}{4} \div \dfrac{x^2 - 4}{6}$

1.7 Expansion Of Three Linear Brackets

To expand three linear brackets you should expand and simplify any two brackets first and then multiply your answer by the third bracket. For example:

14. Expand $(x - 6)(x + 3)(x - 2)$

Expanding two of the brackets, $(x + 3)(x - 2)$, gives:

$x(x - 2) + 3(x - 2) = x^2 - 2x + 3x - 6 = x^2 + x - 6$

Multiplying the answer by the third bracket gives:

$$\begin{aligned}(x - 6)(x^2 + x - 6) &= x(x^2 + x - 6) - 6(x^2 + x - 6)\\ &= x^3 + x^2 - 6x\\ &\quad + \; - 6x^2 - 6x + 36 \quad \text{(adding)}\\ &= x^3 - 5x^2 - 12x + 36\end{aligned}$$

15. Find the values of a, b, c and d for which
$(x-4)(2x+3)(3x-2) = ax^3 + bx^2 + cx + d$

Expanding as before gives:

$(2x+3)(3x-2) = 2x(3x-2) + 3(3x-2)$

$= 6x^2 - 4x + 9x - 6 = 6x^2 + 5x - 6$

Multiplying the answer by the third bracket gives:

$(x-4)(6x^2+5x-6) = x(6x^2+5x-6) - 4(6x^2+5x-6)$

$= 6x^3 + 5x^2 - 6x$

$$\begin{array}{rrrrrl} = & 6x^3 & +\ 5x^2 & -\ 6x & & \\ + & & -\ 24x^2 & -\ 20x & +\ 24 & \text{(adding)} \\ \hline = & 6x^3 & -\ 19x^2 & -\ 26x & +\ 36 & \end{array}$$

So the answer is: a = 6, b = −19, c = −26, d = 24

16. Expand $(x+y)^3$

$(x+y)^3 = (x+y)(x+y)(x+y)$

$(x+y)(x+y) = x(x+y) + y(x+y) = x^2 + xy + xy + y^2$
$= x^2 + 2xy + y^2$

Multiplying the answer by the third bracket gives:

$(x+y)(x^2+2xy+y^2)$

$= x(x^2+2xy+y^2) + y(x^2+2xy+y^2)$

$$\begin{array}{rrrrrl} = & x^3 & +\ 2x^2y & +\ xy^2 & & \\ + & & x^2y & +\ 2xy^2 & +\ y^3 & \text{(adding)} \\ \hline = & x^3 & +\ 3x^2y & +\ 3xy^2 & +\ y^3 & \end{array}$$

Exercise 1G: *Expand the brackets in questions 1 to 12*

1. $(x+1)(x+2)(x+5)$
2. $(x-3)(x+2)(x+4)$
3. $(x-1)(x+4)(x+5)$
4. $(x-4)(x+2)(x-3)$
5. $(x+1)(3x-2)(x+4)$
6. $(2x-1)(3x-2)(x+5)$
7. $(4x+3)(2x+1)(3x-4)$
8. $(x+4)^3$
9. $(x-3)^3$
10. $(2x+5)^3$
11. $(x+2y)^3$
12. $(x-3y)(x+y)(x+5y)$
13. Find the values of a, b, c and d for which $(x-6)(2x+1)(5x-2) = ax^3 + bx^2 + cx + d$

Exercise 1G...

14. Find the values of a, b, c and d for which $(3x-4)(x+3)(x-2) = ax^3 + bx^2 + cx + d$
15. Find the values of a, b, c and d for which $(7x-4)(2x+1)(3x-8) = ax^3 + bx^2 + cx + d$

1.8 Applied Questions

Sometimes questions require you to simplify an expression to solve a problem. For example:

17. The sides of a cube are $(4x-7)$ cm long. Find an expression, in its simplest form, for the volume of the cube.

Volume $= (4x-7)^3 = (4x-7)(4x-7)(4x-7)$

$(4x-7)(4x-7) = 4x(4x-7) - 7(4x-7)$

$= 16x^2 - 28x - 28x + 49 = 16x^2 - 56x + 49$

Multiplying the answer by the third bracket gives:

$(4x-7)(16x^2-56x+49)$

$= 4x(16x^2-56x+49) - 7(16x^2-56x+49)$

$$\begin{array}{rrrrrl} = & 64x^3 & -\ 224x^2 & +\ 196x & & \\ + & & -\ 112x^2 & +\ 392x & -\ 343 & \text{(adding)} \\ \hline = & (64x^3 & -\ 336x^2 & +\ 588x & -\ 343)\ \text{cm}^3 & \end{array}$$

18. A car travels at a constant speed of $(x+3)(2x-5)$ m s^{-1} for $(3x+2)$ seconds. Find an expression, in its simplest form, for the distance travelled, in terms of x.

Distance = speed × time $= (x+3)(2x-5)(3x+2)$

$(2x-5)(3x+2) = 2x(3x+2) - 5(3x+2)$

$= 6x^2 + 4x - 15x - 10 = 6x^2 - 11x - 10$

Multiplying the answer by the third bracket gives:

$(x+3)(6x^2-11x-10)$

$= x(6x^2-11x-10) + 3(6x^2-11x-10)$

$$\begin{array}{rrrrrl} = & 6x^3 & -\ 11x^2 & -\ 10x & & \\ + & & 18x^2 & -\ 33x & -\ 30 & \text{(adding)} \\ \hline = & (6x^3 & +\ 7x^2 & -\ 43x & -\ 30)\ \text{m} & \end{array}$$

Exercise 1H

1. The length, breadth and height of a cuboid are $(x + 5)$ cm, $(x - 2)$ cm and $(3x + 4)$ cm. Find an expression, in its simplest form, for the volume of the cuboid.
2. The sides of a cube are $(2x - 1)$ cm long. Find an expression, in its simplest form, for the volume of the cube.
3. A cylinder has base radius $(x - 4)$ cm and perpendicular height $(x + 3)$ cm. Find an expression, in its simplest form, for its volume, in terms of π.
4. The cost of a rectangular plastic sheet is £$(2x - 1)$ per m^2. Its length and breadth are $(x + 4)$ m and $(x - 2)$ m. Find the total cost.
5. The length, breadth and height of a cuboid are $(2x + 3)$ cm, $(x - 5)$ cm and $(x + 2)$ cm. Find an expression, in its simplest form, for the volume of the cuboid.

1.9 More Complex Questions

Some questions require you to simplify an expression which is itself a numerator or denominator. For example:

19. Simplify $\dfrac{(x - 2)(3x + 2)(x - 4) - 3(x^3 - 5x^2 + 4x)}{x^2 + 4x}$

We need to simplify the numerator by expanding the brackets and collecting like terms.

Firstly, expand $(x - 2)(3x + 2)(x - 4)$:

$(3x + 2)(x - 4) = 3x(x - 4) + 2(x - 4) = 3x^2 - 12x - 8x$

$= 3x^2 - 10x - 8$

and then multiply the answer by the third bracket:

$(x - 2)(3x^2 - 10x - 8)$

$= x(3x^2 - 10x - 8) - 2(3x^2 - 10x - 8)$

$= 3x^2 - 10x^2 - 8x$

$$\begin{array}{rrrrrl} = & 3x^3 & - 10x^2 & - 8x & & \\ + & & - 6x^2 & + 20x & + 16 & \text{(adding)} \\ = & 3x^3 & - 16x^2 & + 12x & + 16 & \end{array}$$

Next, expanding $3(x^3 - 5x^2 + 4x)$ gives:

$3x^3 - 15x^2 + 12x$

Then, combine the two parts of the numerator:

$(3x^3 - 16x^2 + 12x + 16) - (3x^3 - 15x^2 + 12x)$ giving:

$3x^3 - 16x^2 + 12x + 16 - 3x^3 + 15x^2 - 12x = -x^2 + 16$

We can then rewrite the expression as:

$$\frac{16 - x^2}{x^2 + 4x}$$

We can then factorise both numerator and denominator to simplify this fully giving:

$$\frac{(4 - x)(4 + x)}{x(x + 4)}$$

We can then cancel out the $(x + 4)$ to get the final simplified answer of:

$$\frac{4 - x}{x}$$

Exercise 1J: *Simplify the following.*

1. $\dfrac{(2x + 1)(x - 2)(x + 3) - 2x(x^2 - 1) + 6x}{3x - 6}$
2. $\dfrac{(x - 4)(x + 2)^2 - x(x^2 - 4)}{x^2 - 2x - 8}$
3. $\dfrac{(2x + 3)^3 - 2x(4x^2 + 15x + 12) - 3}{3x + 3}$
4. $\dfrac{(x - 1)(x + 2)(2x + 1) - 2x^2(x + 1) + 2}{x^2 - 9}$

CHAPTER 2: EQUATIONS

2.1 Quadratic Equations

A quadratic equation is an equation where the highest power of the variable is 2. You can solve quadratic equations by factorising **but only when it can be factorised.** You can also solve quadratic equations by using the quadratic formula. Solving quadratics by factorising or the formula method will not be asked specifically in the examination but is required in answering questions set on other topics.

Factorising

For example, solve the following:

1. $4x^2 - 25 = 0$

You can factorise $4x^2 - 25$ by the difference of two squares to get: $(2x - 5)(2x + 5) = 0$

So either: $2x - 5 = 0$
in which case: $2x = 5$, giving $x = 2\frac{1}{2}$ or 2.5
or $2x + 5 = 0$
in which case: $2x = -5$, giving $x = -2\frac{1}{2}$ or -2.5

2. $4x^2 - 7x = 0$

You can factorise $4x^2 - 7x$ by common factors to get: $x(4x - 7) = 0$

So either: $x = 0$
or $4x - 7 = 0$
in which case: $4x = 7$, giving $x = 1\frac{3}{4}$ or 1.75

3. $2x^2 + 9x - 5 = 0$

You can factorise $2x^2 + 9x - 5$ by quadratic factors to get: $(2x - 1)(x + 5) = 0$

So either: $2x - 1 = 0$
in which case: $2x = 1$, giving $x = \frac{1}{2}$ or 0.5
or $x + 5 = 0$, giving $x = -5$

Exercise 2A: *Solve the following by factorising.*

1. $9x^2 - 16 = 0$
2. $3x^2 - x - 2 = 0$
3. $12x^2 + 42x = 0$
4. $2x^2 - 7x - 15 = 0$
5. $6x^2 - x - 2 = 0$
6. $3x^2 - 10x - 8 = 0$
7. $4x^2 + 17x - 15 = 0$
8. $6x^2 - 14x = 0$
9. $6x^2 + 23x - 18 = 0$
10. $2x^2 - 17x + 21 = 0$
11. $6x^2 + 7x + 2 = 0$
12. $25x^2 - 4 = 0$

Using The Quadratic Formula

The solutions to $ax^2 + bx + c = 0$ are found from substituting the values of a, b and c into the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For example, solve the following, giving your answers correct to 2 decimal places:

4. $2x^2 + x - 11 = 0$

In this case: a = 2, b = 1 and c = −11.

So $$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times (-11)}}{2 \times 2}$$

$$x = \frac{-1 \pm \sqrt{89}}{4}$$

$$x = \frac{-1 + \sqrt{89}}{4} \text{ or } \frac{-1 - \sqrt{89}}{4}$$

$$x = 2.108 \text{ or } -2.608$$

So $x = 2.11$ or -2.61, to 2 decimal places.

5. $3x^2 - 6x + 1 = 0$

In this case: a = 3, b = −6 and c = 1.

So $$x = \frac{6 \pm \sqrt{36 - 4 \times 3 \times 1}}{2 \times 3}$$

$$x = \frac{6 \pm \sqrt{36 - 12}}{6}$$

$$x = \frac{6 \pm \sqrt{24}}{6}$$

$$x = \frac{6 + \sqrt{24}}{6} \text{ or } \frac{6 - \sqrt{24}}{6}$$

$$x = 1.816 \text{ or } 0.184$$

So $x = 1.82$ or 0.18, to 2 decimal places.

Exercise 2B: *Solve the following, giving your answers correct to 2 decimal places.*

1. $x^2 - 6x + 3 = 0$
2. $2x^2 + x - 7 = 0$
3. $3x^2 - 2x - 9 = 0$
4. $x^2 + 9x + 2 = 0$
5. $2x^2 - x - 7 = 0$
6. $3x^2 + 2x - 2 = 0$
7. $5x^2 - x - 8 = 0$
8. $2x^2 + 8x + 2 = 0$
9. $4x^2 + 5x - 3 = 0$
10. $x^2 - 2x - 9 = 0$

2.2 Setting Up And Solving Quadratic Equations

Sometimes a problem requires us to set up and then solve a quadratic equation. For example:

6. The area of a rectangle is 60 cm^2. The perimeter is 34 cm. Form an equation and solve it to find the lengths of the two sides.

Call the two sides x and y. Then we have:

$xy = 60$ (area of rectangle), and

$2x + 2y = 34$ (perimeter of rectangle)

We can rewrite the perimeter equation in terms of y and then substitute this into the area equation:

$2y = 34 - 2x$

$y = 17 - x$ (dividing by 2) giving:

$x(17 - x) = 60$

$17x - x^2 = 60$ which rearranges to give:

$0 = x^2 - 17x + 60$

We can then factorise this or use the formula to find x:

$(x - 5)(x - 12) = 0$

So: $x = 5$ or 12

Substituting for y gives:

$y = 17 - x$

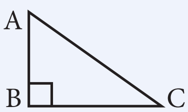
so either $y = 17 - 5 = 12$

or $y = 17 - 12 = 5$

So the lengths of the two sides are 5 cm and 12 cm.

Exercise 2C

1. ABC is a right angled triangle with AC = 15 cm. AB is 3 cm less than BC. Form an equation and solve it to find the lengths of AB and BC.



2. The area of a rectangle is 48 cm^2. The perimeter is 28 cm. Form an equation and solve it to find the lengths of the two sides.
3. Aoife buys x books costing y pence each for £2.80. The cost of the books are then increased by 5p each. She can now buy 2 fewer books for £2.40. Form an equation and solve it to find the initial cost of each book.
4. The width of a rectangle is 5 cm less than its length. The diagonal is 25 cm long. Form an equation and solve it to find the length and width.

2.3 Fractional Equations

For example, solve:

7. $\frac{6}{x-3} + \frac{2}{x-4} = 5$

You need to add the fractions by using the lowest common denominator, which is $(x - 3)(x - 4)$

So we get: $\frac{6(x-4) + 2(x-3)}{(x-3)(x-4)} = 5$

You now need to work out and simplify the numerator:

$6(x - 4) + 2(x - 3) = 6x - 24 + 2x - 6 = 8x - 30$

This gives: $\frac{8x-30}{(x-3)(x-4)} = 5$

You now need to work out and simplify the denominator:

$(x - 3)(x - 4) = x^2 - 7x + 12$

This gives: $\frac{8x-30}{x^2-7x+12} = 5$

You can now cross multiply to get:

$8x - 30 = 5(x^2 - 7x + 12)$

You can now work out the brackets and then put all the terms on one side (with the x^2 term first and positive because it is a quadratic equation):

$8x - 30 = 5x^2 - 35x + 60$

$0 = 5x^2 - 35x - 8x + 60 + 30$

So: $5x^2 - 43x + 90 = 0$

You can now solve this equation, either by factorising or using the quadratic formula, to get:

$(x - 5)(5x - 18) = 0$

So either $x - 5 = 0$, giving $x = 5$

or $5x - 18 = 0$, $5x = 18$ giving $x = \frac{18}{5}$

8. $\frac{4}{x-1} - \frac{2}{1-x} = 3$ where x cannot equal 1

You need to add the fractions by using the lowest common denominator which is $(x - 1)(1 - x)$.

So we get: $\frac{4(1-x) - 2(x-1)}{(x-1)(1-x)} = 3$

You now need to work out and simplify the numerator:

$4(1-x)-2(x-1)=4-4x-2x+2=-6x+6$

This gives: $\dfrac{-6x+6}{(x-1)(1-x)}=3$

You now need to work out and simplify the denominator:

$(x-1)(1-x)=-x^2+2x-1$

This gives: $\dfrac{-6x+6}{-x^2+2x-1}=3$

You can now cross multiply to get:
$-6x+6=3(-x^2+2x-1)$

You can now work out the brackets and then put all the terms on one side (with the x^2 term first and positive because it is a quadratic equation):

$$-6x+6=-3x^2+6x-3$$
$$3x^2-12x+9=0$$

You can divide all these terms by 3 to get:

$$x^2-4x+3=0$$

You can now solve this equation, by factorising or using the quadratic formula, to get:

$(x-3)(x-1)=0$
So either $x-3=0$, giving $x=3$
or $x-1=0$, giving $x=1$

Since we are told in the question that x cannot equal 1 the only solution is $x=3$.

Exercise 2D: *Solve the following.*

1. $\dfrac{4}{x-2}+\dfrac{3}{2x-5}=3$
2. $\dfrac{6}{2x-1}-\dfrac{4}{1-x}=6$
3. $\dfrac{4}{3x-2}-\dfrac{6}{x+1}=1$
4. $\dfrac{2}{x-2}+\dfrac{3}{2x-3}=3$
5. $\dfrac{3}{x+1}-\dfrac{6}{1-2x}=3$
6. $\dfrac{4}{x-3}-\dfrac{4}{1-x}=3$
7. $\dfrac{6}{x-1}-\dfrac{5}{1-2x}=4$
8. $\dfrac{3}{x+1}+\dfrac{6}{2x-1}=3$
9. $\dfrac{4}{x-1}-\dfrac{2}{2-x}=4$
10. $\dfrac{3}{x-2}-\dfrac{2}{3-x}=2$

2.4 Fractional Equations With Both Numerator And Denominator Expressions Involving x

For example, solve:

9. $\dfrac{x+1}{x-1}+\dfrac{3x-1}{2x+1}=4$

You need to add the fractions by using the lowest common denominator which is $(x-1)(2x+1)$.

So we get: $\dfrac{(x+1)(2x+1)+(3x-1)(x-1)}{(x-1)(2x+1)}=4$

You now need to work out and simplify the numerator:

$(x+1)(2x+1)=2x^2+3x+1$ and

$(3x-1)(x-1)=3x^2-4x+1$

So the numerator is:

$2x^2+3x+1+3x^2-4x+1=5x^2-x+2$

This gives: $\dfrac{5x^2-x+2}{(x-1)(2x+1)}=4$

You now need to work out and simplify the denominator:

$(x-1)(2x+1)=2x^2-x-1$

This gives: $\dfrac{5x^2-x+2}{2x^2-x-1}=4$

You can now cross multiply to get:

$5x^2-x+2=4(2x^2-x-1)$

You can now work out the brackets and then put all the terms on one side (with the x^2 term first and positive, as it is a quadratic equation):

$$5x^2-x+2=8x^2-4x-4$$
$$0=8x^2-5x^2-4x+x-4-2$$

So: $3x^2-3x-6=0$

You can divide all these terms by 3 to get:

$$x^2-x-2=0$$

You can now solve this equation, by factorising or using the quadratic formula, to get:

$(x-2)(x+1)=0$

So either $x-2=0$, giving $x=2$

or $x+1=0$, giving $x=-1$