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CCEA GCSE

MATHEMATICS
M8 EXTENSION BOOK

ANSWERS

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v1

Chapter 1: Indices

Exercise 1A

1. (a) 216 (b) $\frac{1}{8}$ (c) 1 (d) $\frac{1}{25}$
2. (a) 1 (b) $\frac{1}{z}$ (c) $\frac{1}{q^2}$ (d) $\frac{1}{w^5}$
3. (a) 13 (b) 2 (c) 3 (d) 2
4. (a) $64p^3$ (b) $243q^5$ (c) $\frac{121}{225}$ (d) $\frac{x^3}{8y^3}$
5. (a) p^7 (b) t^{11} (c) w^{11} (d) x^{16} (e) y^{13}
6. (a) m^4 (b) r^9 (c) p^8 (d) 1 (e) $\frac{1}{y^9}$
7. (a) z^{18} (b) x^{14} (c) p^{45} (d) w^{21} (e) y^{28}

Exercise 1B

1. (a) 11 (b) 7 (c) 10 (d) 5 (e) 2
2. (a) $2a$ (b) $\frac{2}{3}y$ (c) $6z$ (d) $5p$ (e) $\frac{4}{3}q$
3. (a) $z^{\frac{1}{2}}$ (b) $p^{\frac{1}{3}}$ (c) $x^{\frac{1}{2}}$ (d) $y^{\frac{1}{4}}$ (e) $q^{\frac{1}{5}}$
4. (a) $5t^{\frac{1}{2}}$ (b) $2p$ (c) $2y^{\frac{1}{4}}$ (d) $13x$ (e) $5q^{\frac{1}{3}}$
5. (a) $1\frac{1}{4}$ (b) 0 (c) $4\frac{1}{5}$ (d) 6 (e) $-\frac{1}{5}$

Exercise 1C

1. (a) $m^{-\frac{13}{3}}$ (b) Can't simplify (c) $w^{\frac{1}{2}}$ (d) $x^{\frac{23}{4}}$ (e) y^{-13}
2. (a) $q^{\frac{11}{2}}$ (b) r^{15} (c) $p^{-\frac{1}{6}}$ (d) $s^{\frac{9}{20}}$ (e) y^{-1}
3. (a) z^{-18} (b) $x^{\frac{14}{5}}$ (c) p^6 (d) w^{21} (e) y^{-2}
4. (a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $p^{-\frac{13}{4}}$ (d) -4 (e) $-\frac{3}{14}$
5. (a) $45x^{-1}$ (b) $\frac{25}{32y^8}$ (c) x^9
6. (a) 125 (b) $\frac{1}{343}$ (c) 32 (d) $\frac{1}{1000}$ (e) 64
7. (a) $\frac{9}{5}$ (b) $\frac{9}{25}$ (c) $\frac{27}{64}$ (d) $\frac{16}{9}$ (e) $\frac{8}{27}$

Chapter 2: Rational and Irrational Numbers

Exercise 2A

1. **(a)** 7 **(b)** $7\frac{3}{4}$

2. **(a)** $\frac{6}{25}$ **(b)** $\frac{3}{4}$

3. **(a)** 2 **(b)** $\frac{10}{3}$

4. **(a)** 0.666... **(b)** 0.454545...

Exercise 2B

1. **(a)** $2\frac{7}{50}$ **(b)** $3\frac{57}{100}$ **(c)** $\frac{19}{1000}$ **(d)** $-33\frac{51}{5000}$

2. **(a)** $25.\dot{6}$ **(b)** $39.\dot{0}\dot{9}$ **(c)** $6.\dot{1}3\dot{5}$ **(d)** $-467.8\dot{4}6\dot{7}$

3. **(a)** $\frac{7}{9}$ **(b)** $\frac{8}{11}$ **(c)** $\frac{1}{30}$

(d) $\frac{9}{110}$ **(e)** $6\frac{1}{9}$ **(f)** $24\frac{23}{30}$

4. **(a)** $1\frac{5}{333}$ **(b)** $3\frac{23}{99}$ **(c)** $7\frac{425}{999}$

(d) $2\frac{8}{225}$ **(e)** $4\frac{643}{660}$ **(f)** $123\frac{41}{333}$

Exercise 2C

1. **(a)** Irrational **(b)** Rational (= 5) **(c)** Irrational **(d)** Rational (= 3)

2. **(a)** No **(b)** Rational (= 7)

3. **(a)** Rational (= 4) **(b)** Irrational

4. **(a)** $x = \pm\frac{3}{5}$, rational fractions

(b) $x = \pm 2$, integers **(c)** $x = \pm\sqrt{12}$, irrational numbers

5. One possible answer: Add $(1 - \pi)$

6. **(a)** Irrational **(b)** Rational **(c)** Irrational

7. Possible answers: **(a)** $\sqrt{5}$ **(b)** π **(c)** $\sqrt{20}$

8. **(a)** Rational areas: B and C **(b)** Irrational perimeters: A and B

9. Irrational are: π^2 $\frac{\sqrt{27}}{\sqrt{2}}$ $\frac{\sqrt{27}}{\sqrt{4}}$

10. Triangle A has a perpendicular height of $\sqrt{10^2 - 8^2} = 6$

Its area is $\frac{1}{2} \times 8 \times 6 = 24$ square units, which is a rational number.

Triangle B has a perpendicular height of $\sqrt{10^2 - 7^2} = \sqrt{51}$

Its area is $\frac{1}{2} \times 7 \times \sqrt{51} = \frac{7}{2}\sqrt{51}$ square units, which is an irrational number.

Chapter 3: Surds

Exercise 3A

1. **(a)** 49 **(b)** 8 **(c)** 30 **(d)** 5

2. **(a)** $6 + 2x$ **(b)** $19 + 37a - 2a^2$ **(c)** $6 - 6b^2$

3. **(a)** 7.2 cm **(b)** 12 cm^2

4. **(a)** 6 **(b)** $2\sqrt{15}$ **(c)** $2\sqrt{5}$ **(d)** $5\sqrt{3}$ **(e)** $\sqrt{10}$
(f) 2 **(g)** $6\sqrt{14}$ **(h)** 4 **(i)** $21\sqrt{2}$

5. **(a)** $\sqrt{7}$ cm **(b)** 4 cm **(c)** $\sqrt{85}$

6. **(a)** $24\sqrt{3} + 30$ **(b)** $11 + 8\sqrt{2}$ **(c)** $17 - 12\sqrt{2}$

7. $x = \pm 2\sqrt{7}$

Exercise 3B

1. **(a)** $\frac{\sqrt{2}}{2}$ **(b)** $\frac{4\sqrt{5}}{5}$ **(c)** $\frac{\sqrt{6}}{4}$
(d) $\frac{7\sqrt{3}}{15}$ **(e)** $\frac{\sqrt{2}}{4}$ **(f)** $\frac{6\sqrt{10}}{5}$

2. **(a)** $10\sqrt{3}$ **(b)** $\frac{10}{3}\sqrt{6}$ **(c)** $2\sqrt{2}$

3. **(a)** $\frac{3\sqrt{2} + \sqrt{14}}{2}$ **(b)** $2 - \sqrt{2}$ **(c)** $\frac{5\sqrt{2} + 12\sqrt{5}}{5}$

4. $2w^2 = \frac{1}{4}$ or equivalent, leading to $w = \frac{\sqrt{2}}{4}$

5. **(a)** $\pm\frac{5\sqrt{7}}{7}$ **(b)** $\sqrt{2}$ **(c)** 0 or $\pm\frac{3\sqrt{2}}{4}$ **(d)** $\pm\frac{3\sqrt{2}}{2}$

6. $4\sqrt{2}$ m

7. $\frac{5}{4}$

8. **(a)** $7\sqrt{2}$ **(b)** $(4\sqrt{7} + 2\sqrt{2})$ cm

Chapter 4: Simultaneous Equations

Exercise 4A

1. $x = \frac{10}{3}, y = \frac{4}{3}$
2. $x = 3, y = \frac{3}{2}$
3. $x = 5, y = 4$
4. $x = 6, y = 10$

Exercise 4B

1. **(a)** $x = -2$ when $y = 8$ and $x = 3$ when $y = 3$
   (b) $x = -1$ when $y = 7$ and $x = 5$ when $y = -5$
   (c) $x = -1$ when $y = 2$ and $x = 2$ when $y = 8$
   (d) $x = -2$ when $y = 6$ and $x = 4$ when $y = -6$
   (e) $x = 0$ when $y = 3$ and $x = 3$ when $y = 6$
   (f) $x = -3$ when $y = 6$ and $x = 0$ when $y = 3$
2. **(a)** $x = -4, y = 5$ and $x = 5$ and $y = -4$
   (b) $x = -2, y = -6$ and $x = 2, y = 6$
   (c) $x = -4, y = -2$ and $x = 2$ and $y = 4$
   (d) $x = -3, y = 5$ and $x = 5$ and $y = -3$
   (e) $x = -4, y = 4$ and $x = 4$ and $y = -4$
   (f) $x = 3, y = 4$ and $x = -5, y = 0$
   (g) $x = -20, y = -15$ and $x = 7$ and $y = 24$
   (h) $x = -2, y = -6$ and $x = 6, y = 2$

Exercise 4C

1. **(a)** $x = -4.5, y = -2.5$ and $x = 2, y = 4$
   (b) $x = -4, y = 2$ and $x = 4, y = -2$
   (c) $x = 1, y = 2$ and $x = 2, y = 1$
   (d) $x = -7, y = 4$ and $x = 3, y = -4$
2. $x + y = 9$ and $x^2 - y^2 = 45$
   Jack is 7 and Jill is 2 (or vice versa)
3. **(a)** $x = -6, y = -6$ and $x = -0.2, y = 5.6$
   (b) $x = -5, y = -2$ and $x = 7, y = 4$
   (c) $x = -18, y = -2$ and $x = -1.5, y = 3.5$
   (d) $x = -3, y = 1$ and $x = -2, y = 2$
4. **(a)** $x = -2.93, y = -1.93$ and $x = 1.93, y = 2.93$
   (b) $x = -1, y = -3$ and $x = 0.8, y = 2.4$
   (c) $x = -1.21, y = -2.42$ and $x = 1.38, y = 2.75$
   (d) $x = -1.85, y = -2.85$ and $x = 4.85, y = 3.85$
5. **(a)** $x = -3.19, y = -2.19$ and $x = 2.19, y = 3.19$
   (b) $x = 4.26, y = 6.38$ and $x = -4.70, y = -7.05$
   (c) $x = -3.31, y = -2.31$ and $x = 2.31, y = 3.31$
   (d) $x = -3.83, y = -2.83$ and $x = 1.83, y = 2.83$
6. The two numbers are 5.71 and 9.71

Progress Review

Progress Review (Chapters 1–4)

1. **(a)** x^{10} **(b)** y^7 **(c)** t^6
(d) q^3 **(e)** m^{15} **(f)** n^{12}

2. **(a)** 4 **(b)** $11p^2$ **(c)** $5y^3$ **(d)** $9xy^2$

3. **(a)** $\frac{3}{2}$ **(b)** $2\frac{1}{4}$ **(c)** $\frac{6}{49}$ **(d)** 2

4. **(a)** 2 **(b)** $15\frac{5}{8}$

5. **(a)** 2 **(b)** $\frac{5}{7}$

6. **(a)** Rational **(b)** Irrational **(c)** Irrational
(d) Rational **(e)** Rational **(f)** $2.\dot{4} = \frac{22}{9}$ which is rational

7. **(a)** $0.\dot{6}$ **(b)** $0.1\dot{6}$ **(c)** $0.\dot{4}$ **(d)** $0.\dot{4}\dot{5}$

8. **(a)** $\frac{2}{9}$ **(b)** $\frac{8}{11}$ **(c)** $1\frac{14}{99}$ **(d)** $203\frac{17}{90}$

9. **(a)** There are many possible answers, for example $\sqrt{30}$ or $2 + \pi$
(b) There are many possible answers, for example $\sqrt{40}$ or 2π

10. **(a)** There are many possible answers, for example $10 + \sqrt{2}$
(b) There are many possible answers, for example $10(2 + \sqrt{2})$

11. **(a)** $\pm\sqrt{23}$ irrational **(b)** ± 5 integer **(c)** 3 integer

12. **(a)** Area = 60 cm^2, a rational number (and an integer).
(b) Perimeter = $(22 + \sqrt{244})$ cm, an irrational number.

13. Yes. Edge is 6 cm, a rational number.

14. **(a)** $2\sqrt{11}$ **(b)** $4\sqrt{5}$ **(c)** $4\sqrt{7}$ **(d)** $12\sqrt{3}$

15. **(a)** $15 + 4\sqrt{3}$ **(b)** $6 + 4\sqrt{2}$ **(c)** $4 + 3\sqrt{6}$ **(d)** $20 + 10\sqrt{5} - 2\sqrt{15} - 5\sqrt{3}$

16. **(a)** $\sqrt{3}$ **(b)** $5\sqrt{5}$ **(c)** $\sqrt{5}$ **(d)** $9\sqrt{2}$

17. **(a)** $\frac{5\sqrt{3}}{3}$ **(b)** $\frac{5\sqrt{2}}{3}$ **(c)** $\frac{5\sqrt{2} - \sqrt{6}}{2}$

18. **(a)** $\sqrt{135}$ or $3\sqrt{15}$ cm **(b)** $23 - \frac{15}{2}\sqrt{2}$ cm^2

19. **(a)** (2, 6) or (–2, –6) **(b)** (6, 1) or (–6, –1)

20. **(a)** (4, 5) or (–4, –5) **(b)** (6, 6) or (–6, –6)

21. **(a)** (3, 1) or (–3, –1) **(b)** (2, 3) or (–2, –3)

22. **(a)** (–2, 0) or (4, 2) **(b)** (0, 2) or (2, –4)

23. **(a)** (–2.5, 1.5) or (2, 0) **(b)** (–1, 4) or (2.5, 0.5)

24. **(a)** (–2.54, –0.54) or (3.54, 5.54) **(b)** (–1.82, –0.82) or (0.82, 1.82)

25. **(a)** $y - x = 7$ and $xy = 44$
(b) Their ages are 4 and 11 (neglect any negative solutions).

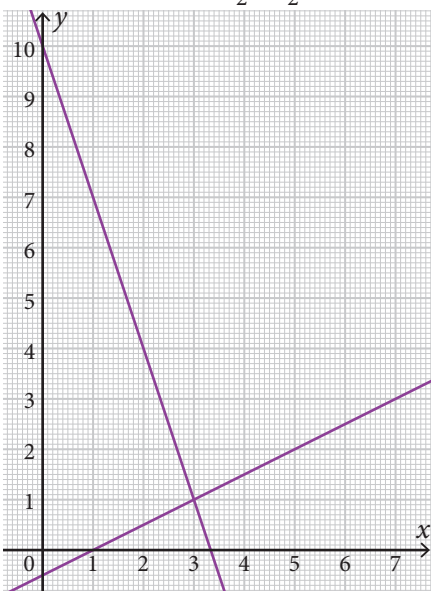
Chapter 5: Graphical Solutions

Exercise 5A

1. (a) $x = 3, y = 1$

(b) (i) $y = -3x + 10$ $\quad y = \frac{1}{2}x - \frac{1}{2}$

(ii)

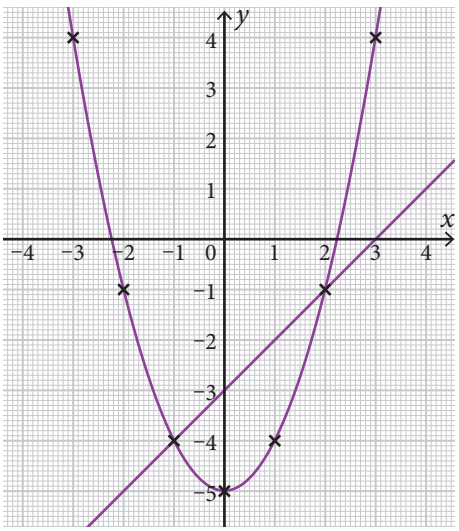


(iii) $x = 3, y = 1$

2. (a)

x	−3	−2	−1	0	1	2	3
y	4	−1	−4	−5	−4	−1	4

(b) – (c)



(d) (−1, −4) and (2, −1)

Exercise 5B

1. **(a)** $x = 2, y = 1$ $x = -2, y = 5$

(b) $x = -2.1, y = 8$ $x = 2.6, y = 8$

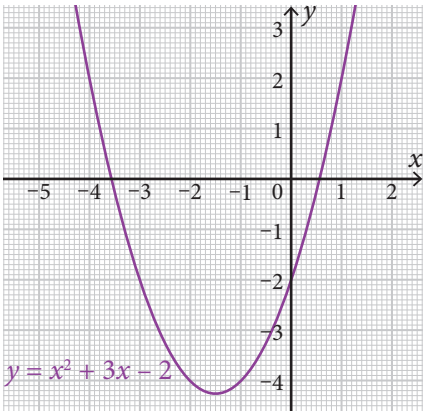
(c) $x = 1.3, y = 1.7$ $x = -3.8, y = -0.9$

(d) $x = 4.2, y = 13.2$ $x = -2.2, y = 6.8$

2. **(a)**

x	−4	−3	−2	−1	0	1
y	2	−2	−4	−4	−2	2

(b)



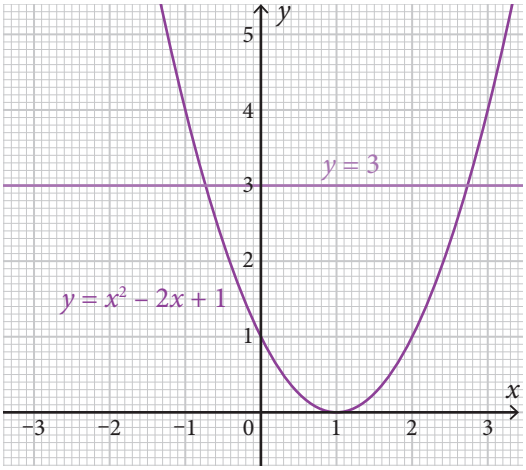
(c) $x = 0.4, y = -0.6$ $x = -2.4, y = -3.4$

Exercise 5C

1. (a)

x	−1	0	1	2	3
y	4	1	0	1	4

(b) – (c)



$x = -0.7, 2.7$

2. (a) $x = -3, 2$

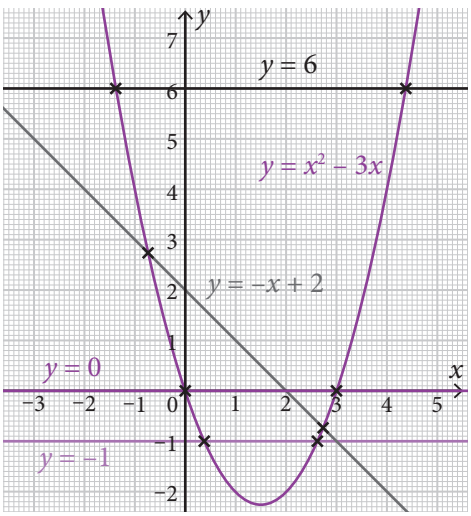
(b) $x = -0.8, 3.8$

3. (a) $x = -2.4, 2.4$

(b) $x = -1.6, 2.6$

(c) $x = -1.2, 1.7$

4. (a)



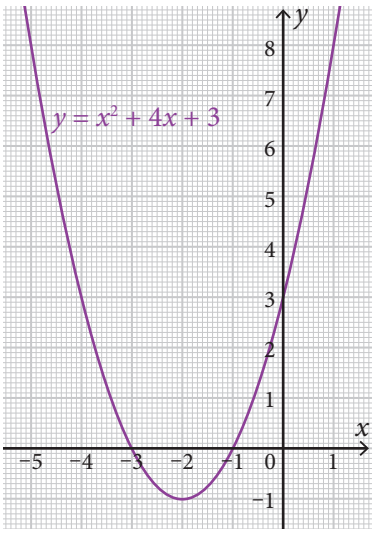
(b) (i) $x = 0, 3$

(ii) $x = -1.4, 4.4$

(iii) $x = 0.4, 2.6$

(iv) $x = -0.7, 2.7$

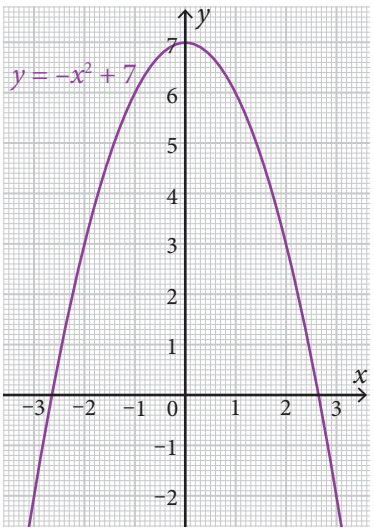
5. **(a)**



(b) **(i)** $x = -2.6, -0.4$

(ii) $x = -4.8, -0.2$

6. **(a)**



(b) **(i)** Line: $y = -\frac{1}{2}x + 2$ $x = -2, 2.5$

(ii) Line: $y = \frac{2}{3}x$ $x = -3, 2.3$

(iii) Line: $y = x + 4$ $x = -2.3, 1.3$

7. **(a)** $y = 6x$

(b) $y = -1.5$

(c) $y = 1$

(d) $y = -4x + 1$

8. **(a)** $x^2 + 6x - 7 = 0$

(b) $x = -7, x = 1$

9. $y = -18x - 16$

10. $y = \frac{3}{4}x - \frac{7}{4}$

Chapter 6: Non-Linear Graphs

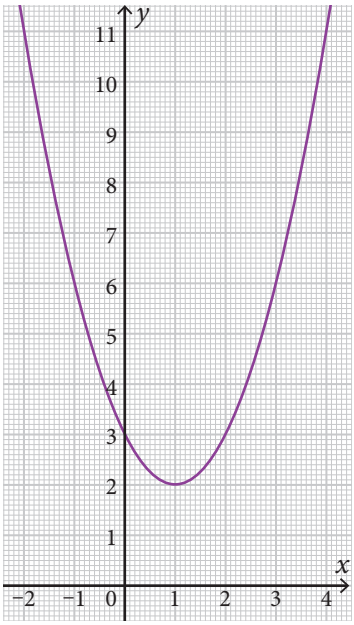
Exercise 6A

1. 1.25

2. **(a)**

x	−2	−1	0	1	2	3	4
y	11	6	3	2	3	6	11

(b)



(c) (1, 2)

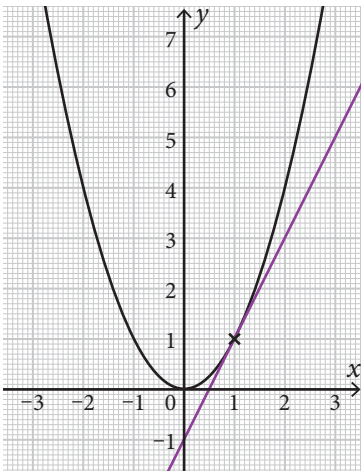
3. **(a)** 3.4×10^{6}

(b) 1.989×10^{3}

(c) 2.251×10^{5}

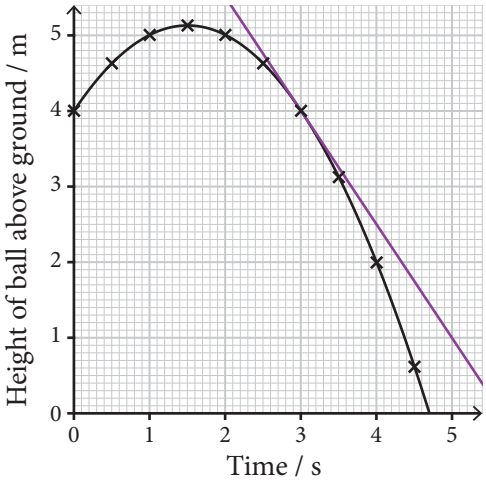
Exercise 6B

1. (a)



(b) 2

2. (a)

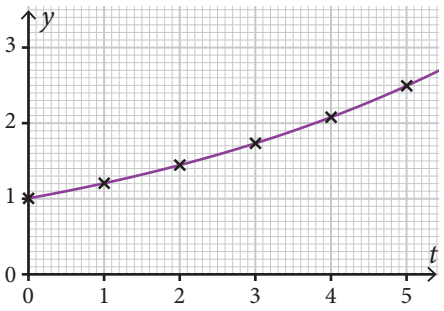


(b) −1.5

(c) The ball's height is changing at a rate of 1.5 metres per second. The negative gradient tells us that the ball is falling rather than rising at this rate.

Exercise 6C

1. (a)



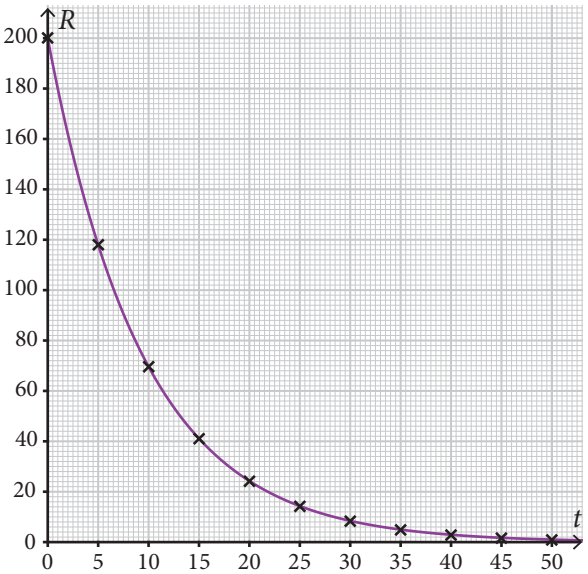
(b) $t = 3.8$

(c) 0.3

2. (a)

t	0	10	20	30	40	50
R	200	69.7	24.3	8.5	3.0	1.0

(b)



(c) 6.5 years

(d) −2.6

3. $a = 3$, $b = 1.3$, $c = \frac{1}{3}$

4. (a) 80

(b) 110

(c) 14

(d) Seema's colony grows faster. The curve is steeper.

(e) 22 bacteria per minute (accept 20 to 25)

(f) 10 minutes

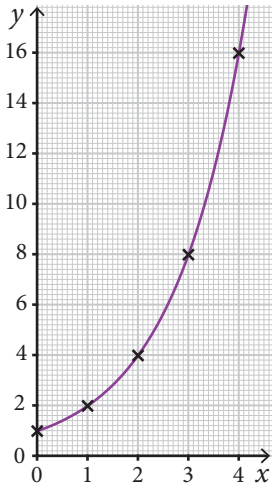
(g) Roughly 9 to 10 minutes

(h) Roughly another 9 minutes

(i) Exponential growth

5. **(a)** 80 °C
(b) 12 minutes
(c) 16 °C
(d) 64 °C
(e) Roughly 3 minutes
(f) Another 3 minutes
(g) Exponential decay
(h) Cooling at roughly 6 °C per minute

6. **(a)**



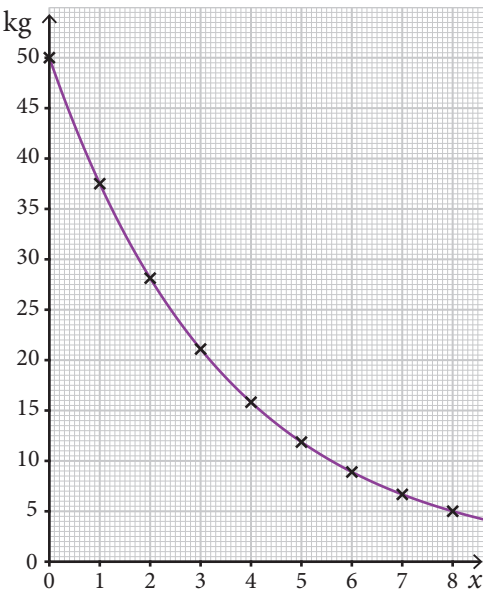
(b) **(i)** £1 **(ii)** The price doubles every year.

(c) 2.8 (accept 2.5 to 3)

7. **(a)**

x	0	1	2	3	4	5	6	7	8
M	50	**37.5**	28.1	**21.1**	15.8	**11.9**	8.9	**6.7**	5.0

(b)



(c) 2.4 years (accept 2.1 to 2.7)

(d) –3.4 kg per year (Accept values between –3 and –3.9)
Accept 'decreasing by', 'reducing by' etc, in place of minus sign.

Exercise 6D

1. **(a)** £1030
   (b) 1.03
   (c) £1194.05
2. 6.6 m/s
3. **(a)** 11 times
   (b) 4 096 000
4. **(a)** $\frac{3}{4}$
   (b) 81
5. **(a)** 0.8
   (b) 2023: 20 480
   2024: 16 384
   (c) 2026
6. $6 \times (1.33333...)^{25} = 7973$
7. **(a)** 171.2 cm
   (b) 81
8. **(a)** 400, 320, 256
   (b) 864
9. **(a)** 90.4 g
   (b) Plastic A: Amount after n years $= 100 \times (0.98)^n$
   Plastic B: Amount after n years $= 120 \times (0.96)^n$
   By trial and improvement, $n = 9$ years when amount B is less than amount A
10. **(a)** 12 hours
    (b) 6.4×10^{10} m
    (c) No. At some point the growth of the star will slow down and stop.

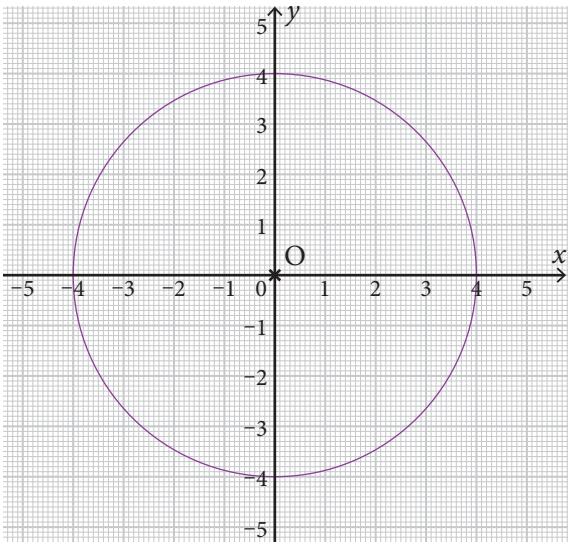
Chapter 7: Circles

Exercise 7A

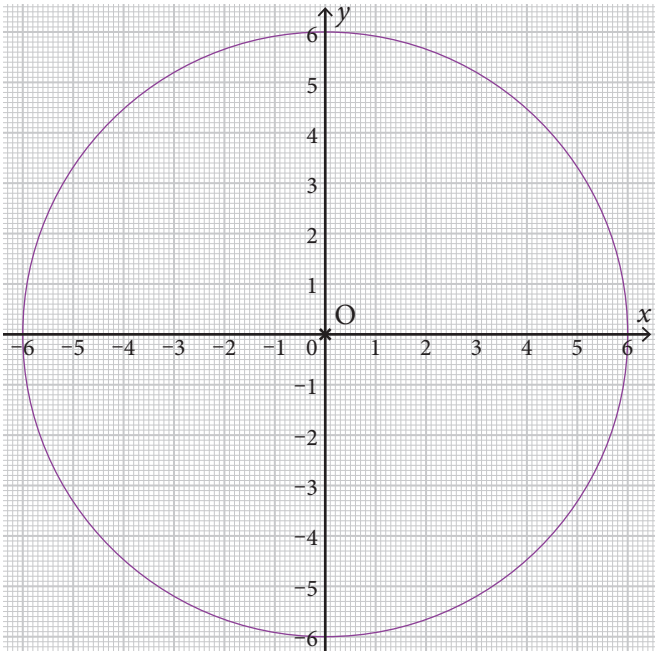
1. **(a)** 45° **(b)** 34° **(c)** 90°
2. **(a)** $5, -\frac{1}{5}$ **(b)** $-2, \frac{1}{2}$ **(c)** $-\frac{2}{5}, \frac{5}{2}$
3. **(a)** $y = 6x$ **(b)** $y = 5x - 6$ **(c)** $y = x - 5$
4. **(a)** (3, 4) and (–4, –3)
   (b) (1, 7) and (–1, –7)
   (c) (5, 3) and (–3, –5)

Exercise 7B

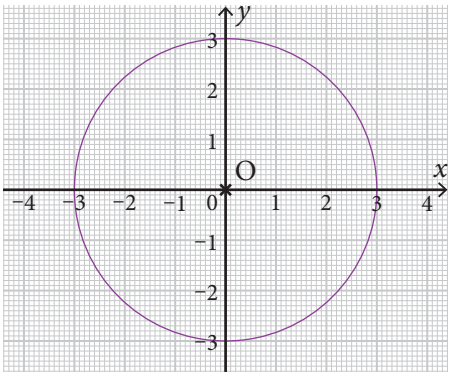
1. **(a)** A circle centred on (0, 0) of radius 7 units
   (b) A circle centred on (0, 0) of radius 9 units
   (c) A circle centred on (0, 0) of radius $\sqrt{20}$ units
2. **(a)** $x^2 + y^2 = 81$ **(b)** $x^2 + y^2 = 7$ **(c)** $x^2 + y^2 = 0.25$
3. **(a)**



3. (b)



(c)



4. (a) The point (5, 12) must satisfy the equation of the circle, so substitute:
$5^2 + 12^2 = 13^2$
$25 + 144 = 169$
$169 = 169$, so the point (5, 12) lies on the circle.

(b) The point (3, 5) must satisfy the equation of the circle, so substitute:
$3^2 + 5^2 = 34$
$9 + 25 = 34$
$34 = 34$, so the point (3, 5) lies on the circle.

(c) The point $(\sqrt{2}, \sqrt{3})$ must satisfy the equation of the circle, so substitute:
$(\sqrt{2})^2 + (\sqrt{3})^2 = 5$
$2 + 3 = 5$
$5 = 5$, so the point $(\sqrt{2}, \sqrt{3})$ lies on the circle.

5. (a) 4 or −4

(b) 9 or −9

(c) 5 or −5

(d) $\sqrt{33}$ or $-\sqrt{33}$

Exercise 7C

1. $y = -\frac{3}{4}x + \frac{25}{4}$
2. $y = \frac{1}{2}x - 5$
3. $y = \frac{3}{2}x - \frac{13}{2}$
4. $y = \frac{4}{3}x + \frac{50}{3}$
5. $y = -\frac{7}{6}x + \frac{85}{6}$
6. $\frac{15}{8}$, $y = -\frac{8}{15}x + \frac{289}{15}$
7. **(a)** 5 or −5 **(b)** $y = x + 10$ or $y = -x + 10$
8. $y = -\frac{\sqrt{2}}{4}x + \frac{9\sqrt{2}}{2}$
9. **(a)** Substitute the points M(5, 12) and N(12, − 5) into the equation of the circle $x^2 + y^2 = 169$
   Both sides evaluate to 169, showing that they satisfy the equation of the circle.
   (b) $\frac{12}{5}$ and $-\frac{5}{12}$
   (c) The lines OM and ON are perpendicular to each other, so $\hat{MON} = 90°$
   And $\hat{MON}$ equals 2 times $\hat{MAN}$ as the angle at centre of circle equals twice the angle at the circumference standing on the same arc. $\hat{MAN} = 45°$
10. **(a)** 8
    (b) Drawing a sketch, the line OP has gradient −1 so $\hat{POQ} = 45°$ and $\hat{POQ}$ = twice $\hat{PRQ}$, using the same property of a circle as in question 9, so $\hat{PRQ} = 22.5°$
11. **(a)** $\sqrt{41}$ **(b)** $y = -\frac{4}{5}x + \frac{41}{5}$ **(c)** $y = -\frac{4}{5}x - \frac{41}{5}$

Exercise 7D

1. (2, 6) and (−2, −6)
2. **(a)** (4, −4) and (−4, 4) **(b)** 90°
3. **(a)** (1, 7) and (7, 1) **(b)** $\sqrt{72}$ or $6\sqrt{2}$
4. **(a)** $y = 3x$ **(b)** (4, 12) and (−4, −12)
5. **(a)** $-\frac{4}{3}$ **(b)** $y = -\frac{4}{3}x$ **(c)** $(3\sqrt{3}, -4\sqrt{3})$ or $(-3\sqrt{3}, 4\sqrt{3})$
6. **(a)** $y = -2x$ **(b)** (−2, 4) **(c)** $x^2 + y^2 = 20$

Chapter 8: Proportion and Variation

Exercise 8A

1. 1200

2. **(a)** $y = kx$ **(b)** 3.25 **(c)** 13 **(d)** 16

Exercise 8B

1. **(a)** $X = \frac{90}{Y}$ (or $Y = \frac{90}{X}$) **(b)** 10 **(c)** 45

2. 60 seconds

3. **(a)** 720 **(b)** 20 researchers

4. **(a)** 140 **(b)** $a = 28$, $b = 0.5$

5. -15, 2

Exercise 8C

1. **(a)** $Y = \frac{500}{X^2}$ **(b)** 5 **(c)** 2.5

2. **(a)** $t = \frac{500}{n^2}$ **(b)** 3 staff

3. $\frac{1}{4}$

4. Statement (2): A is indirectly proportional to the square of B

5. **(a)** Case (1) **(b)** Case (3) **(c)** Case (4)

6. Curve A – Case (4) Curve B – Case (3) Curve C – Case (2) Curve D – Case (1)

7. Statement 1: Curve C
Statement 2: Curve B
Statement 3: Curve A

8. y is halved (or divided by 2)

9. $n = 3 + 3\sqrt{2}$ or $n = 3 - 3\sqrt{2}$

10. **(a)** $w = 6$, $z = 10$

(b) w is inversely proportional to the square root of y, so $w = \frac{k}{\sqrt{y}}$
When $y = 4$, $w = 6$
$6 = \frac{k}{\sqrt{4}}$, giving $k = 6 \times \sqrt{4} = 12$
So $w = \frac{12}{\sqrt{y}}$

(c) w is directly proportional to z^3, so $w = cz^3$
When $w = 6$, $z = 10$
$6 = c \times 10^3$, giving $c = \frac{6}{1000} = \frac{3}{500}$
So $w = \frac{3}{500}z^3$

(d) $\frac{3}{4}$

(e) 5

(f) $w : z = \frac{3}{4} : 5 = 3 : 20$

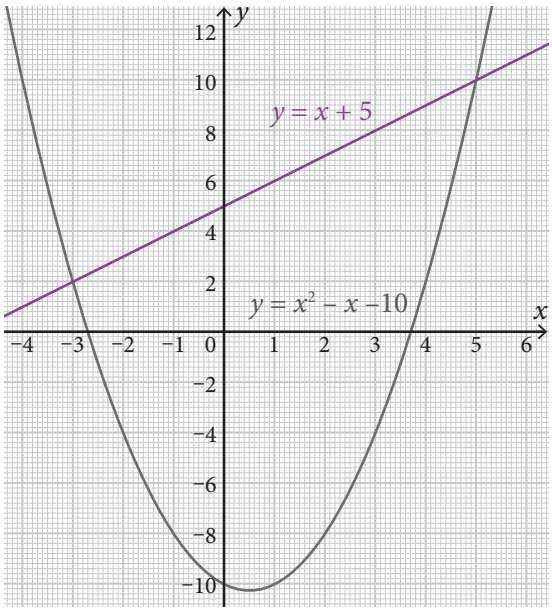
Progress Review

Progress Review (Chapters 5–8)

1. **(a)**

x	−4	−3	−2	−1	0	1	2	3	4	5
y	10	2	**−4**	−8	−10	**−10**	−8	−4	**2**	10

(b) – (c)

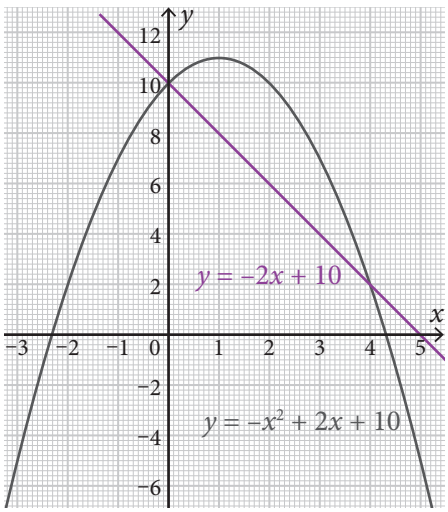


(d) $x = -3, y = 2$
or $x = 5, y = 10$

2. **(a)**

x	−3	−2	−1	0	1	2	3	4	5
y	**−5**	2	7	10	**11**	10	7	**2**	−5

(b) – (c)

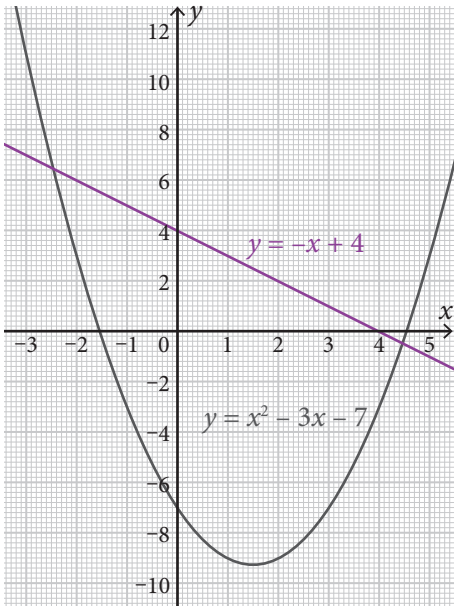


(d) $x = 0, y = 10$
or $x = 4, y = 2$

3. **(a)**

x	−3	−2	−1	0	1	2	3	4	5
y	**11**	3	−3	**−7**	−9	**−9**	−7	−3	**3**

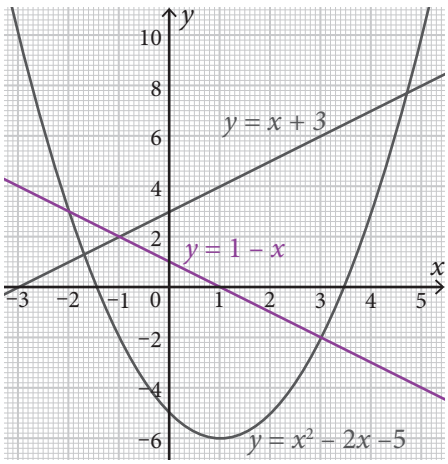
(b) – (c)



(d) $x = 4.5$, $y = -0.5$
or $x = -2.5$, $y = 6..5$

4. **(a)** $x = 4.7$, $y = 7.7$
or $x = -1.7$, $y = 1.3$

(b) Line added: $y = 1 - x$
Solutions: $x = -2$, $y = 3$
or $x = 3$, $y = -2$



5. **(a)** $y = 2x + 1$ **(b)** $y = -2x - 3$ **(c)** $y = 9x - 5$ **(d)** $y = -x - 2.5$ **(e)** $y = -1$

6. Correct answer $-\frac{2}{3}$, but accept answers between −1 and −0.5 exclusive.

7. **(a)** £512.50 **(b)** 1.025 **(c)** £565.70

8. **(a)** 5000 **(b)** 1344 **(c)** 14 June
(d) No, the number of bees does not fall below 640
(e) Falling by 460 bees per day. Accept answers falling by 400 to 500. There should be some indication that the rate of change is negative, e.g. falling, decreasing, or a negative value.

9. **(a)** 2400 **(b)** 2015 **(c)** 15%
(d) This is exponential growth, since the fish population increases by the same percentage each year.
(e) No, this exponential growth could not continue for 20 years because of the limited food resources and space in the lake.
(f) 11165 (accept 11000 to 11500)
(g) Increasing by 776 fish per year (accept 700 to 850)

10. **(a)** 0.123 mg per ml (3 s.f.)
(b) $0.2 \times 0.85^{10} = 0.03937$ mg, where t =10 hours, so time is after 7 pm

11. $8^2 + 3^2 = 73$

12. 9 or –9

13. $x^2 + y^2 = 144$

14. (5, –6)

15. Gradient of OP = $-\frac{5}{12}$ Equation of tangent: $5y = 12x - 169$

16. $5y = 3x + 34$

17. $4y = 25 - 3x$

18. (a) For F: $8^2 + 15^2 = 64 + 225 = 289$ For G: $15^2 + (-8)^2 = 225 + 64 = 289$

(b) Gradient of OF = $\frac{15}{8}$ Gradient of OG = $-\frac{8}{15}$

(c) Product of gradients = $-\frac{8}{15} \times \frac{15}{8} = -1$ so $\hat{FOG} = 90°$

So by circle theorem (the angle at the centre of a circle is equal to twice the angle at the circumference standing on the same arc) angle $\hat{FHG} = 45°$

19. (a) $r^2 = 8^2 + 7^2 = 113$ so $r = \sqrt{113}$

(b) Other tangent is at (–8, –7). Gradient of tangent = $-\frac{8}{7}$

(c) $8x + 7y + 113 = 0$

20. (a) $4y - 3x = 50$

(b) A($-16\frac{2}{3}$, 0)

(c) B(0, $12\frac{1}{2}$) so area = $\frac{1}{2} \times 12\frac{1}{2} \times 16\frac{2}{3} \times 2 = 208\frac{1}{3}$ square units

21. (a) $y = \frac{25}{x}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{5}{3}$

22. P is doubled (or multiplied by 2)

23. (a) $T = \frac{1}{\sqrt{n}}$ **(b)** 10 minutes

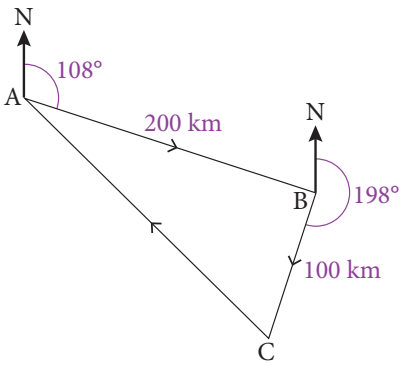
24. (a) $p = \frac{600}{\sqrt{q}}$ **(b)** 1200 **(c)** 4

25. Relationship **A**: y is indirectly proportional to x

Chapter 9: Trigonometry Rules

Exercise 9A

1. $x = \sqrt{410}$ cm (or 20.2 cm), $y = 11$ cm
2. **(a)** 3.56 cm **(b)** 2.35 m
3. 48.6°
4. **(a)** 3 cm^2 **(b)** 10.5 cm^2
5. **(a)**



(b) 90°
(c) 223.6 km, bearing 315°

Exercise 9B

1. **(a)** 2.4 cm **(b)** 2.3 cm **(c)** 5.9 cm **(d)** 20.1 cm
2. **(a)** 41.9° **(b)** 53.5° **(c)** 24.5° **(d)** 44.6°
3. **(a)** 113° **(b)** 104.7 m **(c)** 47.6 m (accept 47.5 m)
4. **(a)** 225° **(b)** 15° **(c)** 135° **(d)** 30° **(e)** 4.14 km
   (f) $\frac{AL}{\sin 135°} = \frac{AB}{\sin 30°}$ Since AB = 8, this gives AL = 11.31 km **(g)** 2.93 km
5. 10.8 m (accept 10.7 m)

Exercise 9C

1. **(a)** 6.1 cm **(b)** 7.5 m **(c)** 4.5 km **(d)** 11.4 cm
2. **(a)** 40.8° **(b)** 41.0° **(c)** 53.8° **(d)** 82.8°
3. 44.4°, 57.1°, 78.5°
4. **(a)** 29.8 m **(b)** 55.1°
5. **(a)** 21.8 m, 31.2 m **(b)** 10.9 m

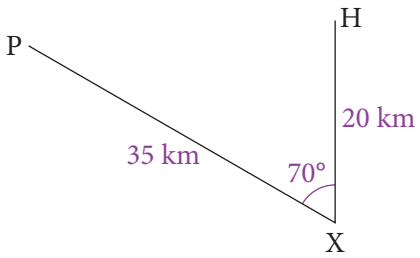
Exercise 9D

1. **(a)** 12.9 cm^2 **(b)** 120 000 m^2 **(c)** 1.17 km^2 **(d)** 1.44 cm^2
2. 3.78 cm
3. 5 cm^2
4. 26.7°
5. $x = -1$ and -15, but triangle side lengths cannot be -10 and -4, so answer $x = -1$
6. 35.8 cm^2 or $\frac{100 + 25\sqrt{3}}{4}$ cm^2

Exercise 9E

1. 73.1 cm^2

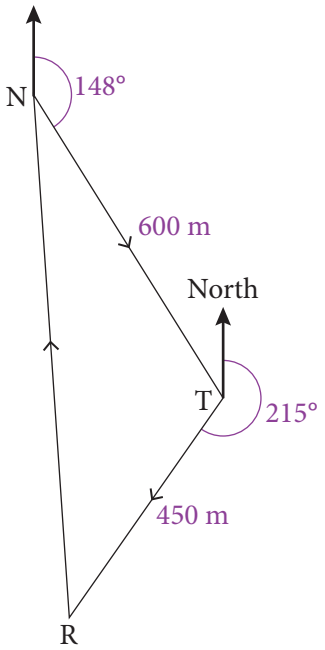
2. **(a)**



(b) 34 km

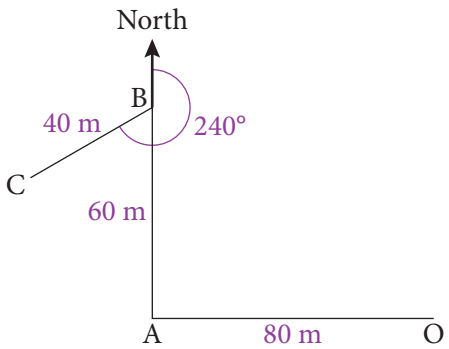
(c) 256° (accept 255°)

3. **(a)**



(b) 113° **(c)** 879.5 m **(d)** 38.9° **(e)** 356°

4. **(a)**



(b) **(i)** 36.9° **(ii)** 53.1° **(iii)** 60° **(iv)** 113.1°

(c) 121.4 m

(d) 49.3°

(e) 109°

5. **(a)** $\frac{1}{2}pq \sin\theta$ **(b)** $\frac{1}{2}qr \sin\theta$

(c) We are told that: $\text{Area ABC} = \frac{3}{4}(\text{Area ACD})$

So: $\frac{1}{2}pq \sin\theta = \frac{3}{4}(\frac{1}{2}qr \sin\theta)$

Divide both sides by $q \sin\theta$: $\frac{1}{2}p = \frac{3}{8}r$

Multiply both sides by 8: $4p = 3r$

Or: $4\text{BC} = 3\text{AD}$

(d) 5 cm

Chapter 10: Pythagoras' Theorem and Trigonometry

Exercise 10A

1. **(a)** $4\sqrt{5}$ cm **(b)** 10 **(c)** 63.4°

2. **(a)** 4.9 cm **(b)** 5.3 cm

Exercise 10B

1. **(a)** 10 cm **(b)** $5\sqrt{5}$ or 11.2 cm **(c)** 26.6°

2. **(a)** 51 m **(b)** 11.3°

3. **(a)** 9.8 cm **(b)** $\sqrt{106}$ or 10.3 cm **(c)** 16.9°

4. $\sqrt{62}$

5. **(a)** $\sqrt{3}$ cm **(b)** 35.3°

6. **(a)** 8.1 m **(b)** 14.2° **(c)** 9.8 m **(d)** 11.7°

7. 29.9°

8. **(a)** 2.68 m **(b)** 5.36 m **(c)** 3.79 m

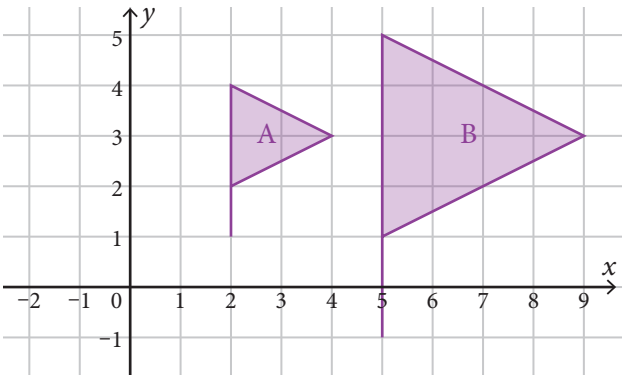
9. **(a)** 4 cm **(b)** $4\sqrt{2}$ cm **(c)** 45°

10. **(a)** **(i)** $\sqrt{221}$ **(ii)** $\sqrt{277}$ **(iii)** $\sqrt{106}$
(b) 37.6°

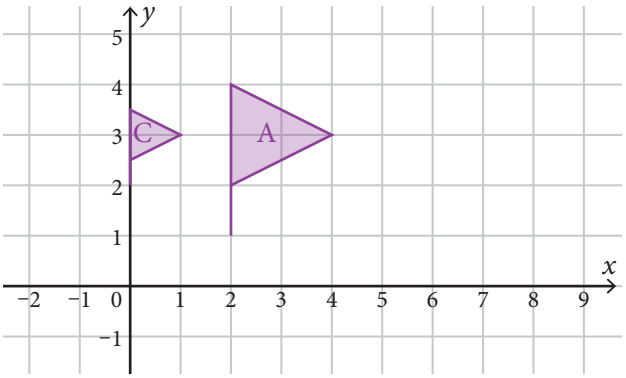
Chapter 11: Enlargement and Similarity

Exercise 11A

1. (a)



(b)

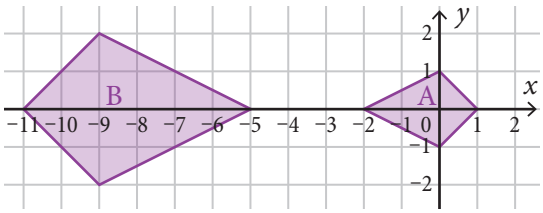


2. (a) 80 cm **(b)** 760 cm^2

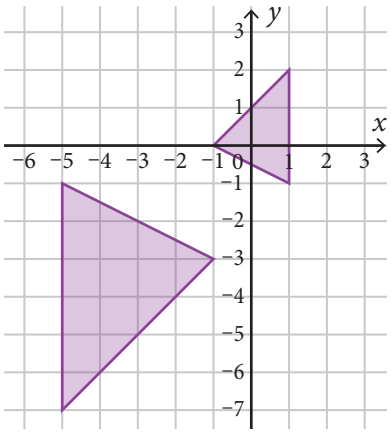
3. Abi's model is 2 times taller than James', so the length scale factor is 2.
The volume scale factor is 2^3, or 8
To calculate the volume of Abi's model, multiply the volume of James' model by 8:
$8 \times 150 = 1200$ cm^3

Exercise 11B

1.



2.

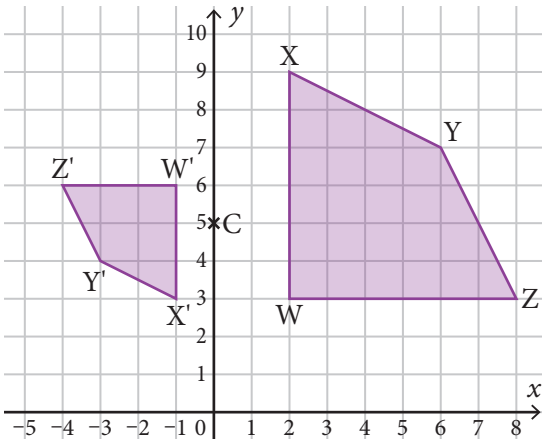


3. No. Congruent shapes have the same size and shape. After an enlargement with a scale factor −2, the sides of the image would be twice as long as the original.

4. Scale factor −3, centre of enlargement (0, 3)

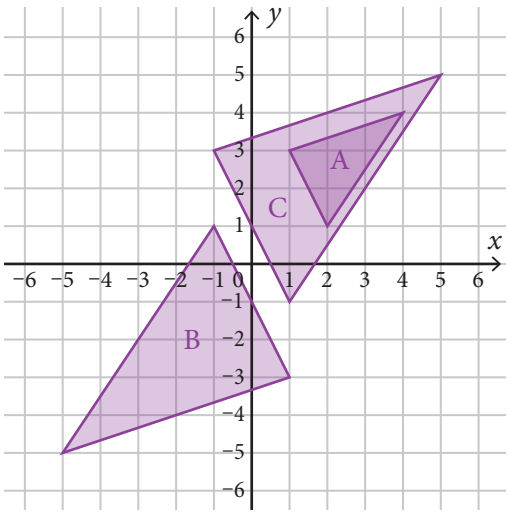
5. Enlargement centre (−1, 0), scale factor −2

6.



7. Enlargement, scale factor −4, centre of enlargement (1, 0)

8. **(a)** – **(c)**



(d) Enlargement, scale factor ½, centre (3, 3)

Exercise 11C

1. **(a)** 880 cm^2 **(b)** 200 cm^3

2. 9600 cm^3

3.

	Shape A	Shape B
Length	3 cm	6 cm
Surface area	10 cm^2	40 cm^2
Volume	25 cm^3	200 cm^3

4. **(a)** 2880π cm^2 **(b)** 10 500π cm^3

5. **(a)** 80 pints **(b)** 360 cm^2 **(c)** 65 cm

6. **(a)** 8 cm **(b)** 216π cm^3

7. **(a)** 40 cm^2 **(b)** 1200 cm^3

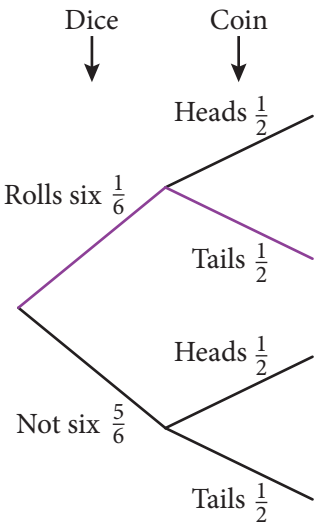
8. 108 g

Chapter 12: Probability

Exercise 12A

1. **(a)** $\frac{1}{6}$ **(b)** $P(B) = \frac{3}{6} = \frac{1}{2}$

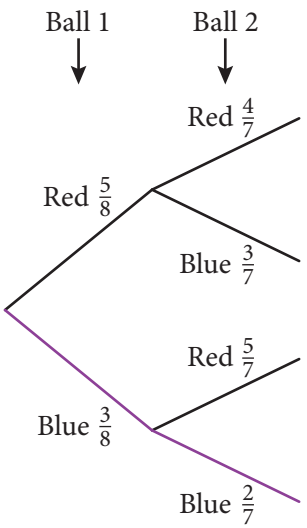
2. **(a)**



(b) $P(6, \text{Tails}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

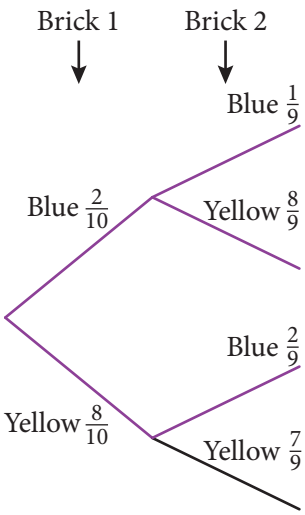
Exercise 12B

1.



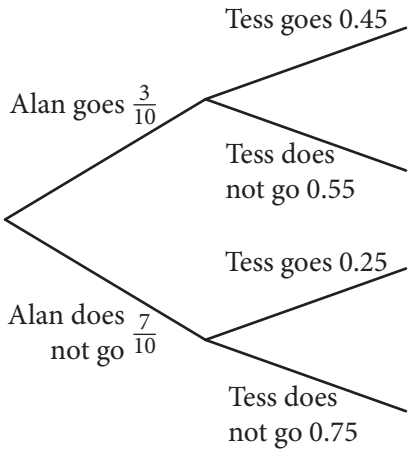
$P(\text{Blue, Blue}) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

2. (a)



(b) (i) $\frac{28}{45}$ **(ii)** $P(B, B) + P(Y, Y) = \frac{2}{10} \times \frac{1}{9} + \frac{8}{10} \times \frac{7}{9} = \frac{2}{90} + \frac{56}{90} = \frac{58}{90} = \frac{29}{45}$

3. (a)

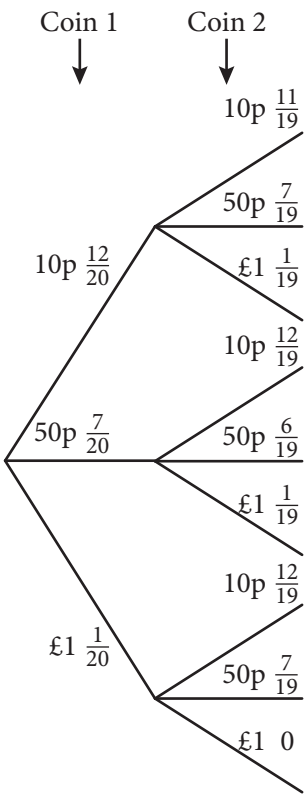


(b) 0.135 or $\frac{27}{200}$

(c) 0.525 or $\frac{21}{40}$

(d) 0.34 or $\frac{17}{50}$

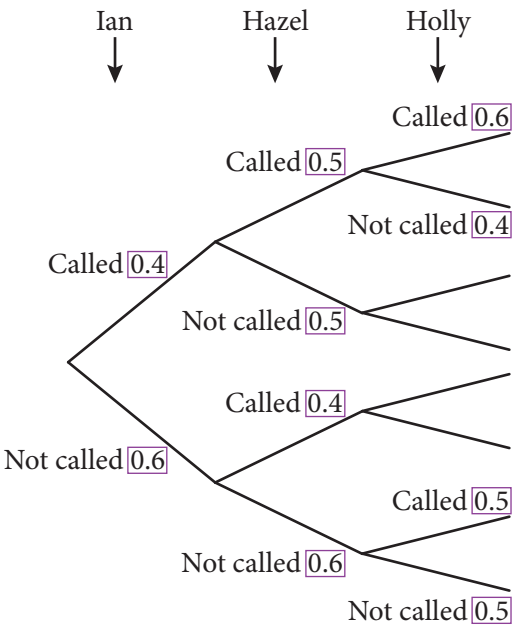
4. (a)



(b) **(i)** 0.347 **(ii)** 0.458 **(iii)** 0.889

(c) 0

5. (a)

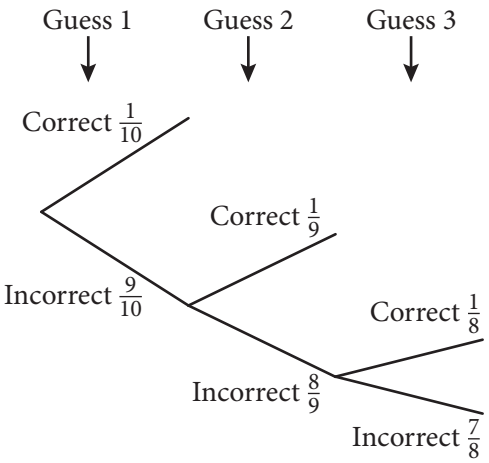


(b) P(All called) = $0.4 \times 0.5 \times 0.6 = 0.12$

P(None called) = $0.6 \times 0.6 \times 0.5 = 0.18$

It is more likely that none of the family members are called out.

6. (a)



(b) Sue wins if she guesses correctly the 1st time OR is incorrect the 1st time and is correct the 2nd time OR is incorrect the 1st and 2nd times and is correct the 3rd time.

$P(\text{Sue wins}) = \frac{1}{10} + \left(\frac{9}{10} \times \frac{1}{9}\right) + \left(\frac{9}{10} \times \frac{8}{9} \times \frac{1}{8}\right) = \frac{3}{10}$

7. (a) (i) $\frac{3}{35}$ or 0.086 (3 d.p.) **(ii)** $\frac{3}{7}$ or 0.429 (3 d.p.)

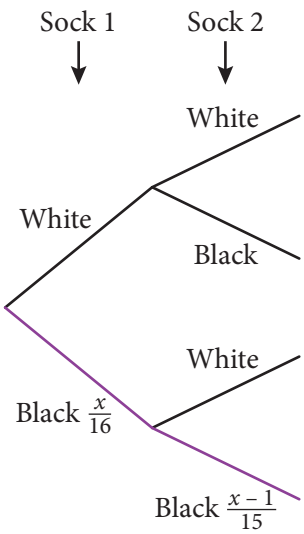
(b) $P(\text{white, white}) = \frac{35}{50} \times \frac{34}{49} = \frac{17}{35}$

$P(\text{at least one red}) = 1 - \frac{17}{35} = \frac{18}{35}$

P(white, white) < P(at least one red), so Philip is correct

8. 0.005 or $\frac{1}{200}$

9. (a)



$\frac{x}{16} \times \frac{x-1}{15} = \frac{3}{8}$

$\frac{x^2 - x}{240} = \frac{3}{8}$

$x^2 - x = 90$

$x^2 - x - 90 = 0$

(b) $(x - 10)(x + 9) = 0$

$x = 10$ or $x = -9$

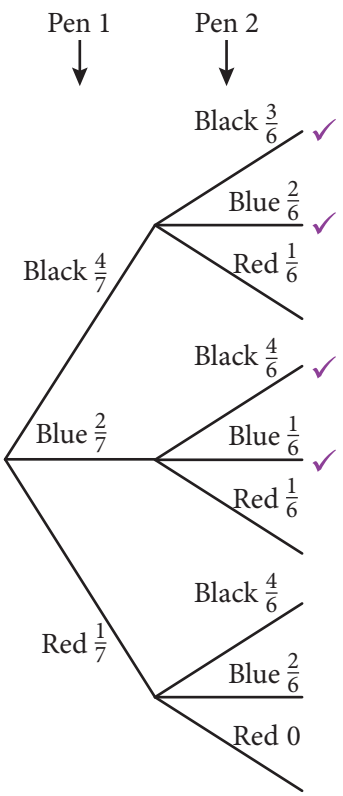
The negative answer does not make sense in the context of the question, so $x = 10$

(c) $P(\text{Black, Black}) = \frac{3}{8}$

$P(\text{White, White}) = \frac{6}{16} \times \frac{5}{15} = \frac{1}{8}$

$P(\text{Same colour}) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$

10. (a)



P(Black, Black) $= \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

P(Blue, Blue) $= \frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$

P(Red, Red) = 0

P(Same colour) $= \frac{2}{7} + \frac{1}{21} = \frac{1}{3}$

(b) The combinations in which neither pen is red are ticked in the tree diagram.

P(Black, Black) $= \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

P(Black, Blue) $= \frac{4}{7} \times \frac{2}{6} = \frac{4}{21}$

P(Blue, Blue) $= \frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$

P(Blue, Black) $= \frac{2}{7} \times \frac{4}{6} = \frac{4}{21}$

P(Neither red) $= \frac{2}{7} + \frac{4}{21} + \frac{1}{21} + \frac{4}{21} = \frac{15}{21}$

11. $\frac{9}{40} \times \frac{8}{39} = \frac{3}{65}$

12. (a) $\frac{5}{15} \times \frac{4}{14} = \frac{2}{21} = 0.095$

(b) $\frac{6}{16} \times \frac{5}{15} = \frac{1}{8} = 0.125$

(c) Probability of 2 red balls $= \frac{x}{(x+10)} \times \frac{(x-1)}{(x+9)} = \frac{1}{2}$

$$x(x-1) = \tfrac{1}{2}(x+10)(x+9)$$
$$2x(x-1) = (x+10)(x+9)$$
$$2x^2 - 2x = x^2 + 10x + 9x + 90$$
$$x^2 - 21x - 90 = 0$$

$x = 24.65$ or -3.65 (cannot have a negative number of balls so reject -3.65)

$x = 25$ since there can only be a whole number of red balls

Exercise 12C

1. $\frac{2}{6} \times \frac{3}{7} + \frac{4}{6} \times \frac{2}{7} = \frac{14}{42} = \frac{1}{3}$

2. Table of values constructed from question:

Club Members	Over 16	Under 16	Total
Boys	7	16	23
Girls	12	15	27
Total	**19**	**31**	**50**

Probability of choosing a member who is a girl under 16 = $\frac{15}{50} = \frac{3}{10}$

3. **(a)** Probability of both yellow = $\frac{8}{10} \times \frac{7}{9} = \frac{56}{90} = \frac{28}{45}$

Probability of selecting red after 2 balls chosen = $1 - \frac{28}{45} = \frac{17}{45}$

(b) Probability of selecting red after 3 balls chosen = $1 - \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} = 1 - \frac{336}{720} = 1 - 0.4666... = 0.533...$

4. **(a)** You may find it helpful to draw a probability table with dice 1 across the top and dice 2 down the side.
Score of 7 with 2 dice = (1, 6), (6, 1), (2, 5), (5, 2), (3, 4) or (4, 3)
There are 36 possible combinations of 2 dice, so probability of a total of 7 = $\frac{6}{36} = \frac{1}{6}$

(b) There are 11 combinations containing a 6. But one of these is (6, 6) so we discount that.
Hence, the probability of getting just one 6 = $\frac{10}{36} = \frac{5}{18}$

5. There are 22 available dates. Sam can attend on 17 of these dates. So the probability = $\frac{17}{22}$

6. Table of values constructed from question:

Guests	Wedding	Not at wedding	Total
Women	45	35	80
Men	42	8	50
Children	13	7	20
Total	**100**	**50**	**150**

6. **(a)** $\frac{35}{150} = \frac{7}{30}$ **(b)** $\frac{13}{20}$

7. **(a)** Let x be the number of pink hats.
Probability of choosing 2 pink hats from 9 hats = $\frac{x}{9} \times \frac{x-1}{8} = \frac{5}{12}$

$$x(x-1) = 30$$
$$x^2 - x - 30 = 0$$
$$(x-6)(x+5) = 0$$
$$x = 6 \text{ or } x = -5$$

We cannot have a negative number of hats, so the answer is 6 pink hats.

(b) Probability of choosing exactly one pink hat = $2 \times \frac{6}{9} \times \frac{3}{8} = \frac{1}{2}$

8. Use a Tree diagram
P(Red, Red) = $0.4 \times 0.45 = 0.18$
P(Red, Not Red) = $0.4 \times 0.55 = 0.22$
P(Not Red, Red) = $0.6 \times 0.35 = 0.21$
P(Not Red, Not Red) = $0.6 \times 0.65 = 0.39$ or $1 - 0.18 - 0.22 - 0.21 = 0.39$

9. 10 digits available so numbers 000 through to 999 = 1000 numbers but digits are all different.
There are 10 choices for 1st digit, 9 choices for 2nd digit and 8 choices for 3rd digit.
Total choices = $10 \times 9 \times 8 = 720$ different numbers where all 3 digits are different.
Therefore probability of guessing the code = $\frac{1}{720}$

10. One possible example:

Suppose there are 5 marbles, 2 of them red.

Using selection with replacement, the probability of choosing two reds is $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25} = 0.16$

Using selection without replacement, the probability of choosing two reds is $\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = 0.1$

There is a greater probability of choosing two red marbles using selection with replacement.

Progress Review

Progress Review (Chapters 9–12)

1. 7.1 m

2. 69.1°

3. 28.1 cm^2

4. 6.13 mm

5. 63.2°

6. 24.7°

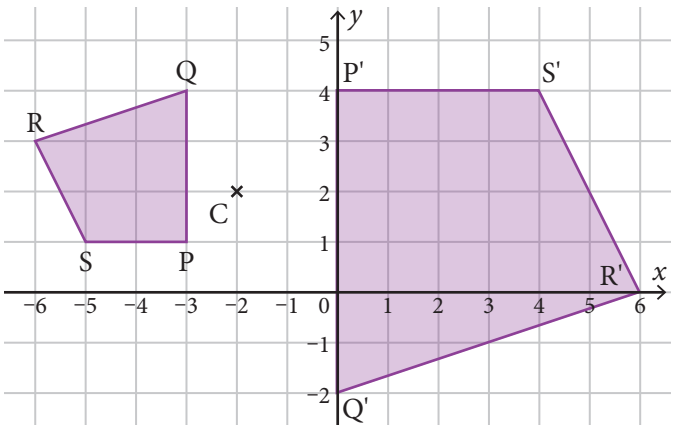
7. **(a)** 11.2 cm **(b)** 13.2 cm **(c)** 32.1°

8. $\sqrt{248}$ or $2\sqrt{62}$

9. 1.5°

10. 31.8 m

11.



12. Enlargement scale factor –3, centre of enlargement (0, 3)

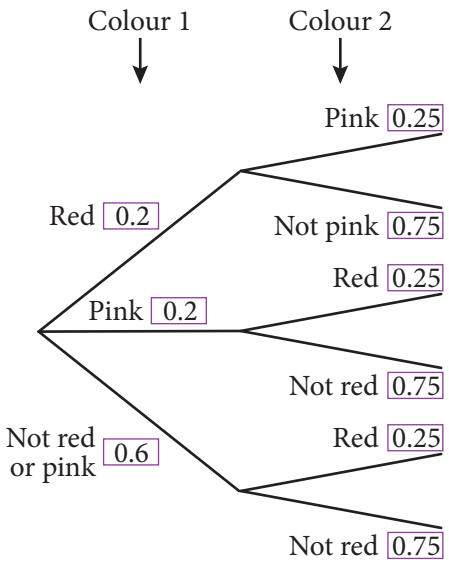
13. 506.25π cm^3

14.

	Shape A	Shape B
Width	5 m	**15 m**
Base area	8 m^2	72 m^2
Volume	12 m^3	**324 m^3**

15. **(a)** 640 cm^3 **(b)** 40 cm^2

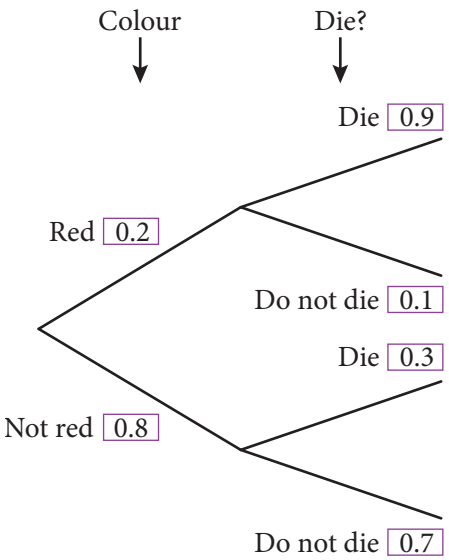
16. (a)



(b) (i) P(Red, Pink) = 0.2 × 0.25 = 0.05
P(Pink, Red) = 0.2 × 0.25 = 0.05
P(Red and Pink) = 0.05 + 0.05 = 0.1

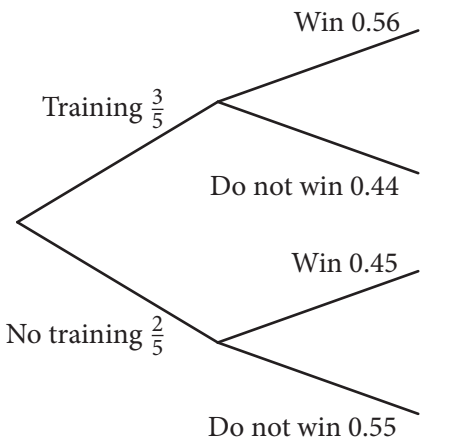
(ii) P(Red, Pink) = 0.2 × 0.25 = 0.05
P(Red, Not pink) = 0.2 × 0.75 = 0.15
P(Pink, Red) = 0.2 × 0.25 = 0.05
P(Not pink or red, Red) = 0.6 × 0.25 = 0.15
P(Red) = 0.05 + 0.15 + 0.05 + 0.15 = 0.4

(c)



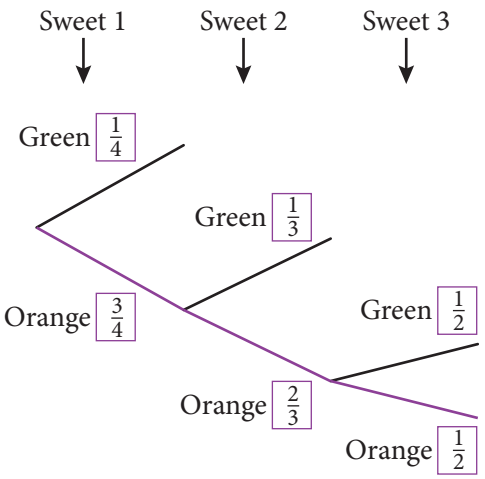
(d) 0.42

17. (a)



(b) 0.516

18. (a)



(b) P(Orange, Orange, Orange) $= \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{4}$

19. (a) Let x be the number of black cards, so there are $2x$ cards in the pack.

$$\text{P(B, B)} = \frac{x}{2x} \times \frac{x-1}{2x-1} = \frac{9}{38}$$

$$\frac{1}{2} \times \frac{x-1}{2x-1} = \frac{9}{38} \quad \text{so} \quad \frac{x-1}{2x-1} = \frac{18}{38}$$

$38(x - 1) = 18(2x - 1)$ giving $x = 10$

So the number of cards = 20

(b) (i) $\frac{x}{x+10} \times \frac{x-1}{x+9} = \frac{60}{119}$

$119 \times x \times (x - 1) = 60 \times (x + 10) \times (x + 9)$

$119(x^2 - x) = 60(x^2 + 19x + 90)$

$59x^2 - 1259x - 5400 = 0$

(ii) $x = 25$, so 15 black cards were added

20. P(late) $= 0.4 \times \frac{1}{10} + 0.6 \times \frac{1}{2} = 0.34$ or $\frac{17}{50}$

21. (a) (i) Probability $= \frac{206}{398} = \frac{103}{199}$ or 0.518 (3 d.p.)

(ii) Number of females taking meal = 270 – 145 = 125

So P(female taking meal) $= \frac{125}{398}$ or 0.314 (3 d.p.)

(b) Total females = 192

Total females not having a meal = 192 – 125 = 67

So P(no meal given that it is a female) $= \frac{67}{192}$ or 0.349 (3 d.p.)